Overview on Classical Logics for KR&R

Knowledge bases

- Knowledge base = set of sentences in a formal language = logical theory
  - explicit statement of sentences believed (including any assumed connections among non-logical symbols)
- Which logic? There are several logics: e.g.
  - Propositional Logic
  - Predicate Logic
  - Logic Programs
  - Description Logics
  - Modal Logics
  - Non-monotonic logic
  - Intuitionistic logic
  - ...
- But, there are “standard approaches”:

  Propositional and Predicate logic

Types of Logic

- Logics are characterized by what they commit to as "primitives"
- Ontological commitment: what exists–facts? objects? time? beliefs?
- Epistemological commitment: what states of knowledge?

Language	Ontological Commitment (What exists in the world)	Epistemological Commitment (What an agent believes about facts)
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief 0…1
Fuzzy logic	degree of truth	degree of belief 0…1

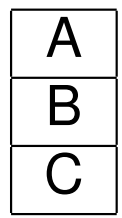
Classical logics are based on the notion of TRUTH

Entailment – Logical Implication

$$KB \models \alpha$$

- Knowledge base *KB* entails sentence α if and only if α is true in all worlds where *KB* is true
- The KB containing “humans will die” and “Socrates is a human” entails “Socrates will die”
- Explicit knowledge: $\beta \in KB$, e.g. “Socrates is a human”
- Implicit knowledge: $\beta \in \{\alpha \mid KB \models \alpha\}$, e.g. “Socrates will die”
- Often non trivial: explicit $\rightarrow$ implicit

Three blocks stacked
Top one is green
Bottom one is red



green

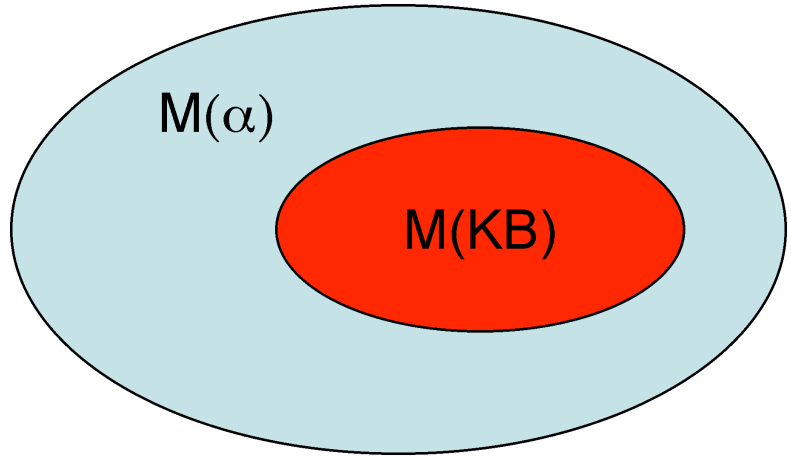
red

Is there a green block directly on top of a non-green block? Yes.

Models

- Logicians typically think in terms of interpretations, which are formally structured worlds with respect to which truth can be evaluated
  - an interpretation *I* assigns meaning (truth) to atomic symbols and specifies how to interpret more complex sentences
- Special attention is devoted to models
  - an Interpretation *I* is a model of a sentence α iff α is true in *I*
- $M(\alpha)$ is the set of models of α

- Then $KB \models \alpha$ if and only if $M(KB) \subseteq M(\alpha)$



  - Note: What if $M(KB) = \emptyset$, i.e. the KB has no models?
  - It that case, we say that *KB* is inconsistent
  - As $\emptyset = M(KB) \subseteq M(\alpha)$, it follows that

$$KB \models \alpha \text{ , for any sentence } \alpha$$

  - That is, anything is entailed by an inconsistent KB

Inference, Deduction, Reasoning

$$\boxed{KB \vdash \alpha}$$

- $KB \vdash \alpha$ = sentence α can be derived/prooved from KB by the procedure $\vdash$
- Soundness: $\vdash$ is sound if

  whenever $KB \vdash \alpha$, it is also true that $KB \models \alpha$
- Completeness: $\vdash$ is complete if

  whenever $KB \models \alpha$, it is also true that $KB \vdash \alpha$