General Concept Inclusions in Fuzzy Description Logics

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Abstract. *Fuzzy Description Logics* (fuzzy DLs) have been proposed as a language to describe structured knowledge with vague concepts. A major theoretical and computational limitation so far is the inability to deal with *General Concept Inclusions* (GCIs), which is an important feature of classical DLs. In this paper, we address this issue and develop a calculus for fuzzy DLs with GCIs.

1 INTRODUCTION

Description Logics (DLs) [2] are a logical reconstruction of the so-called frame-based knowledge representation languages, with the aim of providing a simple well-established Tarski-style declarative semantics to capture the meaning of the most popular features of structured representation of knowledge. Nowadays, DLs have gained even more popularity due to their application in the context of the *Semantic Web*, as the theoretical counterpart of OWL DL (the W3C standard for specifying ontologies, see [9] for details).

Fuzzy DLs [15, 18, 23, 24] extend classical DLs by allowing to deal with *fuzzy/vague/imprecise concepts* such as "Candia is a creamy white rose with dark pink edges to the petals", "Jacaranda is a hot pink rose", and "Calla is a very large, long white flower on thick stalks". Such concepts involve so-called *fuzzy* or *vague concepts*, like "creamy", "dark", "hot", "large" and "thick", for which a clear and precise definition is not possible.

The problem to deal with imprecise concepts has been addressed several decades ago by Zadeh [25], which gave birth in the meanwhile to the so-called *fuzzy set and fuzzy logic theory* and a huge number of real life applications exists. Despite the popularity of fuzzy set theory, relative little work has been carried out in extending DLs towards the representation of imprecise concepts, notwithstanding DLs can be considered as a quite natural candidate for such an extension [1, 3, 4, 5, 6, 11, 12, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 26].

A major theoretical and computational limitation so far of fuzzy DLs is the inability to deal with *General Concept Inclusions* (GCIs), which is an important feature of classical DLs; e.g., GCIs are necessary to represent domain and range constraints. In this paper, we address this issue and develop a calculus for fuzzy DLs with GCIs.

In the next section, we briefly recall basic concepts of DLs and fuzzy DLs, while in Section 3 we present a sound and complete calculus dealing with GCIs. Section 4 concludes.

2 PRELIMINARIES

DLs basics. DLs [2] are a family of logics for representing structured knowledge. Each logic is identified by a name made of labels, which identify the operators allowed in that logic. Major DLs are the so-called logic $\mathcal{ALC}$ [13] and is used as a reference language whenever new concepts are introduced in DLs, $\mathcal{SHOIN}(\mathtt{D})$, which is the logic behind the ontology description language OWL DL and $\mathcal{SHIF}(\mathtt{D})$, which is the logic behind OWL LITE, a slightly less expressive language than OWL DL (see [9]). A DL can be seen as a restricted First Order Language with unary and binary predicates. For the sake of our purpose we deal here with $\mathcal{ALC}$, whose syntax and semantics is described in Table 1, (an *interpretation* $\mathcal{I} = (\Delta^\mathcal{I}, \cdot^\mathcal{I})$ has domain $\Delta^\mathcal{I}$ and maps concepts into subsets of $\Delta^\mathcal{I}$, maps roles into subsets of $\Delta^\mathcal{I} \times \Delta^\mathcal{I}$ and maps individuals into elements of $\Delta^\mathcal{I}$). An $\mathcal{ALC}$ *knowledge base* is defined as a pair $\Sigma = \langle \mathcal{T}, \mathcal{A} \rangle$, where $\mathcal{T}$ is called a *TBox* and $\mathcal{A}$ an *ABox*. $\mathcal{T}$ is a finite set of *general inclusion axioms* (GCIs for short) of the form, $C \sqsubseteq D$ and $\mathcal{A}$ is a finite set of *concept* and *role assertions* of the form $a : C$ and $(a, b) : R$, respectively. For example $\mathcal{T}$ could contain an axioms of the form $\mathtt{HappyFather} \sqsubseteq \exists\mathtt{hasChild}.\mathtt{Female}$, and $\mathcal{A}$ an assertion of the form $Tom : \mathtt{HappyFather}$. An interpretation $\mathcal{I}$ satisfies $\mathcal{T}$ if $C^\mathcal{I} \subseteq D^\mathcal{I}$ for all GCIs in $\mathcal{T}$, then $\mathcal{I}$ is called a *model* of $\mathcal{T}$, and it satisfies $\mathcal{A}$ if $a^\mathcal{I} \in C^\mathcal{I}$ ($\langle a^\mathcal{I}, b^\mathcal{I} \rangle \in R^\mathcal{I}$) for all concept (role) assertions in $\mathcal{A}$. Then $\mathcal{I}$ is called a model of $\mathcal{A}$. An interpretation satisfies an $\mathcal{ALC}$ KB $\Sigma = \langle \mathcal{T}, \mathcal{A} \rangle$ if it satisfies both $\mathcal{A}$ and $\mathcal{T}$; then $\mathcal{I}$ is called a *model* of Σ. A concept C is *subsumed by* a concept D, written $C \sqsubseteq D$, with respect to (w.r.t.) $\mathcal{T}$, if for all models $\mathcal{I}$ of $\mathcal{T}$, it holds that $C^\mathcal{I} \subseteq D^\mathcal{I}$. An ABox $\mathcal{A}$ is *consistent* (*inconsistent*) w.r.t. a TBox $\mathcal{T}$ if there exists (does not exist) a model $\mathcal{I}$ of $\mathcal{T}$, that is also a model of $\mathcal{A}$. Finally, Σ *entails* an $\mathcal{ALC}$ assertion ϕ, written $\Sigma \models \phi$, if each model of Σ is a model of ϕ.

Fuzzy DLs basics. Fuzzy DLs [18] extend classical DLs by allowing to deal with fuzzy/imprecise concepts. The main idea underlying fuzzy DLs is that an assertion $a{:}C$, stating that the constant a is an instance of concept C, rather being interpreted as either true or false, will be mapped to a truth value $n \in [0, 1]_\mathbb{Q}$ (the rationals in the unit interval $[0, 1]$). The intended meaning is that n indicates to which extent 'a is a C'. From a syntax point of view, concepts, roles, individuals and concept inclusion axioms are as for $\mathcal{ALC}$. In place of assertions, we have *fuzzy assertions* [18], which are of the form $\langle \alpha \bowtie n \rangle$, where α is an assertion, $n \in [0, 1]_\mathbb{Q}$ and $\bowtie$ is one of $\geq, \leq, >, <$. For instance, $\langle a{:}C \geq n \rangle$ allows to state that individual a is an instance of concept C at least to degree n. Similarly for role assertions. A *Fuzzy Knowledge Base*, $\Sigma = \langle \mathcal{T}, \mathcal{A} \rangle$, is as for the crisp case, except that now $\mathcal{A}$ is a set of fuzzy assertions rather than assertions only. From a semantics point of view, a *fuzzy interpretation* $\mathcal{I} = (\Delta^\mathcal{I}, \cdot^\mathcal{I})$ has domain $\Delta^\mathcal{I}$, but now maps a concept C into a function $C^\mathcal{I} \colon \Delta^\mathcal{I} \to [0, 1]_\mathbb{Q}$ and a role R into a function $R^\mathcal{I} \colon \Delta^\mathcal{I} \times \Delta^\mathcal{I} \to [0, 1]_\mathbb{Q}$. For $d \in \Delta^\mathcal{I}$, $C^\mathcal{I}(d)$ gives us the degree of d being an instance of the concept C (similarly for roles). The semantics is summarized in Table 1. A fuzzy interpretation $\mathcal{I}$ satisfies a fuzzy TBox $\mathcal{T}$ if $\forall x \in \Delta^\mathcal{I}.C^\mathcal{I}(x) \leq D^\mathcal{I}(x)$ for all

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Table 1. $\mathcal{ALC}$ and fuzzy $\mathcal{ALC}$.

Concepts						
Syntax			**Classical Semantics**			**Examples**
$C, D \longrightarrow$	$\top \mid$	(top concept)	$\top^{\mathcal{I}}$	$=$	$\Delta^{\mathcal{I}}$	
	$\bot \mid$	(bottom concept)	$\bot^{\mathcal{I}}$	$=$	$\emptyset$	
	$A \mid$	(atomic concept)	$A^{\mathcal{I}}$	$\subseteq$	$\Delta^{\mathcal{I}}$	`Human`
	$C \sqcap D \mid$	(concept conjunction)	$(C_1 \sqcap C_2)^{\mathcal{I}}$	$=$	${C_1}^{\mathcal{I}} \cap {C_2}^{\mathcal{I}}$	`Human` $\sqcap$ `Male`
	$C \sqcup D \mid$	(concept disjunction)	$(C_1 \sqcup C_2)^{\mathcal{I}}$	$=$	${C_1}^{\mathcal{I}} \cup {C_2}^{\mathcal{I}}$	`Nice` $\sqcup$ `Rich`
	$\neg C \mid$	(concept negation)	$(\neg C)^{\mathcal{I}}$	$=$	$\Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$	$\neg$`Male`
	$\exists R.C \mid$	(existential quantification)	$(\exists R.C)^{\mathcal{I}}$	$=$	$\{x \mid \exists y.\langle x, y, \in\rangle R^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\}$	$\exists$`has_child.Blond`
	$\forall R.C$	(universal quantification)	$(\forall R.C)^{\mathcal{I}}$	$=$	$\{x \mid \forall y.\langle x, y, \in\rangle R^{\mathcal{I}} \Rightarrow y \in C^{\mathcal{I}}\}$	$\forall$`has_child.Human`

Fuzzy Semantics					
$\top^{\mathcal{I}}(x)$	$=$	1	$(C_1 \sqcup C_2)^{\mathcal{I}}(x)$	$=$	$\max({C_1}^{\mathcal{I}}(x), {C_2}^{\mathcal{I}}(x))$
$\bot^{\mathcal{I}}(x)$	$=$	0	$(\neg C)^{\mathcal{I}}(x)$	$=$	$\neg C^{\mathcal{I}}(x))$
$A^{\mathcal{I}}(x)$	$\in$	$[0,1]_{\mathbb{Q}}$	$(\exists R.C)^{\mathcal{I}}(x)$	$=$	$\sup_{y \in \Delta^{\mathcal{I}}}\{\min(R^{\mathcal{I}}(x,y), C^{\mathcal{I}}(y))\}$
$(C_1 \sqcap C_2)^{\mathcal{I}}(x)$	$=$	$\min({C_1}^{\mathcal{I}}(x), {C_2}^{\mathcal{I}}(x))$	$(\forall R.C)^{\mathcal{I}}(x)$	$=$	$\inf_{y \in \Delta^{\mathcal{I}}}\{\max(1 - R^{\mathcal{I}}(x,y), C^{\mathcal{I}}(y))\}$

$C \sqsubseteq D \in \mathcal{T}$, then $\mathcal{I}$ is called a *model* of $\mathcal{T}$, and it satisfies a fuzzy ABox $\mathcal{A}$ if $C^{\mathcal{I}}(a^{\mathcal{I}}) \bowtie n$ ($R^{\mathcal{I}}(a^{\mathcal{I}}, b^{\mathcal{I}}) \bowtie n$), for each $\langle a : C \bowtie n\rangle$ ($\langle (a,b) : R \bowtie n\rangle$) in $\mathcal{A}$. Then $\mathcal{I}$ is called a *model* of $\mathcal{A}$. A fuzzy $\mathcal{ALC}$ KB $\Sigma = \langle \mathcal{T}, \mathcal{A}\rangle$ is consistent if there exists a model $\mathcal{I}$ of $\mathcal{A}$ and $\mathcal{T}$.

Given a fuzzy KB Σ, and a fuzzy assertion ψ (resp. a GCI $C \sqsubseteq D$), we say that Σ *entails* ψ (resp. $C \sqsubseteq D$), denoted $\Sigma \models \psi$ (resp. $\Sigma \models C \sqsubseteq D$), if each model of Σ is a model of ψ (resp. $C \sqsubseteq D$). Finally, given Σ and a fuzzy assertion α, it is of interest to compute α's best lower and upper truth value bounds. The *greatest lower bound* of α w.r.t. Σ (denoted $glb(\Sigma, \alpha)$) is $glb(\Sigma, \alpha) = \sup\{n \mid \Sigma \models \langle \alpha \geq n\rangle\}$ where $\sup \emptyset = 0$. Similarly, the *least upper bound* of α w.r.t. Σ (denoted $lub(\Sigma, \alpha)$) is $lub(\Sigma, \alpha) = \inf\{n \mid \Sigma \models \langle \alpha \leq n\rangle\}$ where $\inf \emptyset = 1$. Determining the *glb* is called the *Best Truth Value Bound* (BTVB) problem. Basic inference problems are: (i) Check if a fuzzy KB is *consistent*, i.e. has a model. (ii) Check if D *subsumes* C w.r.t. Σ, i.e. $\Sigma \models C \sqsubseteq D$. (iii) Check if a is instance of C to degree $\geq n$, i.e. $\Sigma \models \langle a{:}C \geq n\rangle$ (Similarly for the other relations $\leq, >$ and $<$). (iv) Determine $glb(\Sigma, a{:}C)$.

We recall that all the inference problems can be reduced to the consistency problem [18]: (i) Concerning the entailment problem, it can be verified that it can be reduced to the inconsistency problem: $\langle \mathcal{T}, \mathcal{A}\rangle \models \langle \alpha \geq n\rangle$ iff $\langle \mathcal{T}, \mathcal{A} \cup \{\langle \alpha < n\rangle\}\rangle$ is inconsistent, and similarly for the other relations $\leq, >$ and $<$; (ii) Concerning the BTVB problem, it holds that $lub(\Sigma, a{:}C) = 1 - glb(\Sigma, a{:}\neg C)$, i.e. the lub can be determined through the glb (and vice-versa). Furthermore, the computation of the *glb* can be determined by relying on a finite number of entailment tests. First, for $\Sigma = \langle \mathcal{T}, \mathcal{A}\rangle$, we define $X^{\mathcal{A}} = \{0, 0.5, 1\} \cup \{n \mid \langle \alpha \bowtie n\rangle \in \mathcal{A}\}$ and $N^{\mathcal{A}} = X^{\mathcal{A}} \cup \{1 - n \mid n \in X^{\mathcal{A}}\}$. Then $glb(\Sigma, a{:}C) = \max\{n \mid n \in N^{\mathcal{A}}$ and $\Sigma \models \langle \alpha \geq n\rangle\}$. (iii) Concerning the subsumption problem, we have that $\Sigma \models C \sqsubseteq D$ iff $\Sigma = \langle \mathcal{T}, \{\langle a{:}C \geq n\rangle, \langle a{:}D < n\rangle\}\rangle$, with $n \in \{n_1, n_2\}$, $n_1 \in (0, 0.5]$ and $n_2 \in (0.5, 1]$ (e.g., we may choose $n_1 = 0.25, n_2 = 0.75$), is not consistent. So, the subsumption problem can reduced to the consistency problem as well.

In all previous approaches to fuzzy DLs [18, 6, 14] decision procedures for the consistency, the entailment and the BTVB problem are given for various DL languages, but with restrictions on the form of concept inclusion axioms in a TBox $\mathcal{T}$. More precisely, $\mathcal{T}$ was considered to be *simple*, i.e. cyclic axioms are not allowed, while concept inclusions were restricted to those of the form $A \sqsubseteq D$, where A is an *atomic* concept. Both GCIs and cyclic axioms are considered important for the classical case and, thus, should be provided in the fuzzy variant as well. For instance, cyclic definitions allow us to consider definitions such as

`Human` $\sqsubseteq \exists$`hasParent.Human`
`GeometricElement` $\sqsubseteq \forall$`hasPart.GeometricElement` .

It is also well known that GCIs are used to express important features like: (i) The *domain* of a role R is concept C. This can be expressed by means of the GCI, $\exists R.\top \sqsubseteq C$. (ii) The *range* of a role R is concept C. This can be expressed by means of the GCI, $\top \sqsubseteq \forall R.C$ [5]. (iii) Concept C_1 and concept C_2 are *disjoint*. This can be expressed by means of the GCI, $C_1 \sqcap C_2 \sqsubseteq \bot$.[6] Such features appear in the OWL DL [9] and hence also in the fuzzy OWL DL language [16]. In the following we will present a calculus for fuzzy $\mathcal{ALC}$ with both GCIs and cyclic axioms.

3 DEALING WITH GCIs IN FUZZY DLs

Suppose that we have an individual a and a concept C. Then, for any fuzzy interpretation $\mathcal{I}$, $\forall n \in [0,1]_{\mathbb{Q}}$ either $C^{\mathcal{I}}(a^{\mathcal{I}}) < n$ or $C^{\mathcal{I}}(a^{\mathcal{I}}) \geq n$ holds [7]. Furthermore, if $\mathcal{I} \models C \sqsubseteq D$, then either $C^{\mathcal{I}}(a^{\mathcal{I}}) < n$ or $D^{\mathcal{I}}(a^{\mathcal{I}}) \geq n$. From this observation, the following can be shown.

Proposition 1 *$\mathcal{I} \models C \sqsubseteq D$ iff for all $n \in [0,1]_{\mathbb{Q}}$, either $\mathcal{I} \models \langle a : C < n\rangle$ or $\mathcal{I} \models \langle a : D \geq n\rangle$, for all a.*

This suggests that the models of a TBox $\mathcal{T}$ can be captured in the form of mutually exclusive ABoxes. For example, if $\mathcal{T} = \{C_1 \sqsubseteq D_1, C_2 \sqsubseteq D_2\}$, then for any n, the four alternatives are (i) $\{\langle a{:}C_1 < n\rangle, \langle a{:}C_2 < n\rangle\}$; (ii) $\{\langle a{:}C_1 < n\rangle, \langle a{:}D_2 \geq n\rangle\}$; (iii) $\{\langle a{:}D_1 \geq n\rangle, \langle a{:}C_2 < n\rangle\}$; and (iv) $\{\langle a{:}D_1 \geq n\rangle, \langle a{:}D_2 \geq n\rangle\}$. Note that this is a generalisation of crisp DLs where for any crisp model $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ of $\mathcal{T}$, we have that $\forall d \in \Delta^{\mathcal{I}}.d \in ((\neg C_1 \sqcup D_1) \sqcap (\neg C_2 \sqcup D_2))^{\mathcal{I}}$. Please note that this *internalized* [2] concept cannot be used in fuzzy DLs since a GCI of the form $C \sqsubseteq D$ is not equivalent to the concept $\neg C \sqcup D$. Hence, in order to decide the consistency of a $\Sigma = \langle \mathcal{T}, \mathcal{A}\rangle$, for each individual a that exists in $\mathcal{A}$, or might be created by the reasoning algorithm, we have to create 2^k ABoxes, where k is the number of GCIs that exist in $\mathcal{T}$.

However, it is practically impossible to devise a terminating reasoning algorithm that uses Proposition 1 to handle GCIs and cyclic axioms as we cannot realistically apply it to all $n \in [0,1]_{\mathbb{Q}}$. Fortunately, we can restrict these n to a *finite* set of values. Indeed, from the previous section, it turns out that the good candidate is the set $N^{\mathcal{A}}$. In [18, 19] it is shown that if a fuzzy $\mathcal{ALC}$ ABox is consistent, then there exists a model where the membership degrees used to build the model are restricted to those that exist in the ABox. For instance,

[5] Note that the top concept ($\top$) is not an atomic concept, hence range restrictions indeed are GCIs.
[6] Note that $C \sqsubseteq \neg D$ is not a proper disjoint axiom in fuzzy $\mathcal{ALC}$.
[7] Similarly, either $C^{\mathcal{I}}(a^{\mathcal{I}}) \leq n$ or $C^{\mathcal{I}}(a^{\mathcal{I}}) > n$.

in order to satisfy $\{\langle a{:}C \geq n\rangle\}$, we set $C^{\mathcal{I}}(a^{\mathcal{I}}) = n$, while to satisfy $\{\langle a{:}C > n\rangle\}$, we set $C^{\mathcal{I}}(a^{\mathcal{I}}) = n + \epsilon$, for a sufficiently small $\epsilon \in [0,1]_{\mathbb{Q}}$.

In the following, we assume that an ABox $\mathcal{A}$ has been *normalized*, i.e. fuzzy assertions of the form $\langle a{:}C > n\rangle$ are replaced by $\langle a{:}C \geq n + \epsilon\rangle$ and those of the form $\langle a{:}C < n\rangle$, by $\langle a{:}C \leq n - \epsilon\rangle$. Please note that in a normalized fuzzy KB we allow the degree to range in $[-\epsilon, 1+\epsilon]_{\mathbb{Q}}$ in place of $[0,1]_{\mathbb{Q}}$. It can be proved that the process of normalization is satisfiability preserving.

Proposition 2 *Let $\Sigma = \langle \mathcal{T}, \mathcal{A}\rangle$ be a fuzzy knowledge base. Then Σ is satisfiable if and only if its normalized variant is satisfiable.*

3.1 A fuzzy tableau for fuzzy $\mathcal{ALC}$

We have seen that the inference problems in fuzzy DLs can be reduced to the consistency checking problem. Similar to crisp DLs, our tableaux algorithm checks the consistency of a fuzzy KB by trying to build a fuzzy tableau, from which it is immediate either to build a model in case KB is consistent or to detect that the KB is inconsistent. The fuzzy tableau we present here can be seen as an extension of the tableau presented in [8], and is inspired by the one presented in [14]. Without loss of generality, we assume that all concepts C are in *negation normal form* (NNF) [7], i.e. negations occur in front of atomic concepts only.[8] In the following, $\bowtie \in \{\geq, \leq\}$, while we also use $\bowtie^-$ to denote the *reflection* of $\bowtie$, e.g. if $\bowtie = \leq$, then $\bowtie^- = \geq$.

Definition 1 *Given $\Sigma = \langle \mathcal{T}, \mathcal{A}\rangle$, let $\boldsymbol{R}_\Sigma$ be the set of roles occurring in Σ and let $sub(\Sigma)$ be the set of named concepts appearing in Σ. A* fuzzy tableau *T for Σ is a quadruple ($\boldsymbol{S}$, $\mathcal{L}$, $\mathcal{E}$, $\mathcal{V}$) such that: $\boldsymbol{S}$ is a set of elements, $\mathcal{L} : \boldsymbol{S} \times sub(\Sigma) \to [0,1]_{\mathbb{Q}}$ maps each element and concept, to a membership degree (the degree of the element being an instance of the concept), and $\mathcal{E} : \boldsymbol{R}_\Sigma \times 2^{\boldsymbol{S}\times\boldsymbol{S}} \to [0,1]_{\mathbb{Q}}$ maps each role of $\boldsymbol{R}_\Sigma$ and pair of elements to the membership degree of the pair being an instance of the role, and $\mathcal{V} : \mathbf{I}_{\mathcal{A}} \to \boldsymbol{S}$ maps individuals occurring in $\mathcal{A}$ to elements in $\boldsymbol{S}$. For all $s, t \in \boldsymbol{S}$, $C, E \in sub(\Sigma)$, and $R \in \boldsymbol{R}_\Sigma$, T has to satisfy:*

1. *$\mathcal{L}(s, \bot) = 0$ and $\mathcal{L}(s, \top) = 1$ for all $s \in \boldsymbol{S}$,*
2. *If $\mathcal{L}(s, \neg C) \bowtie n$, then $\mathcal{L}(s, C) \bowtie^- 1 - n$.*
3. *If $\mathcal{L}(s, C \sqcap E) \geq n$, then $\mathcal{L}(s, C) \geq n$ and $\mathcal{L}(s, E) \geq n$.*
4. *If $\mathcal{L}(s, C \sqcup E) \leq n$, then $\mathcal{L}(s, C) \leq n$ and $\mathcal{L}(s, E) \leq n$.*
5. *If $\mathcal{L}(s, C \sqcup E) \geq n$, then $\mathcal{L}(s, C) \geq n$ or $\mathcal{L}(s, E) \geq n$.*
6. *If $\mathcal{L}(s, C \sqcap E) \leq n$, then $\mathcal{L}(s, C) \leq n$ or $\mathcal{L}(s, E) \leq n$.*
7. *If $\mathcal{L}(s, \forall R.C) \geq n$, then $\mathcal{E}(R, \langle s, t\rangle) \leq 1 - n$ or $\mathcal{L}(t, C) \geq n$ for all $t \in \boldsymbol{S}$.*
8. *If $\mathcal{L}(s, \exists R.C) \leq n$, then $\mathcal{E}(R, \langle s, t\rangle) \leq n$ or $\mathcal{L}(t, C) \leq n$ for all $t \in \boldsymbol{S}$.*
9. *If $\mathcal{L}(s, \exists R.C) \geq n$, then there exists $t \in \boldsymbol{S}$ such that $\mathcal{E}(R, \langle s, t\rangle) \geq n$ and $\mathcal{L}(t, C) \geq n$.*
10. *If $\mathcal{L}(s, \forall R.C) \leq n$, then there exists $t \in \boldsymbol{S}$ such that $\mathcal{E}(R, \langle s, t\rangle) \geq 1 - n$ and $\mathcal{L}(t, C) \leq n$.*
11. *If $C \sqsubseteq D \in \mathcal{T}$, then either $\mathcal{L}(s, C) \leq n - \epsilon$ or $\mathcal{L}(s, D) \geq n$, for all $s \in \boldsymbol{S}$ and $n \in N^{\mathcal{A}}$.*
12. *If $\langle a{:}C \bowtie n\rangle \in \mathcal{A}$, then $\mathcal{L}(\mathcal{V}(a), C) \bowtie n$.*
13. *If $\langle (a,b){:}R \bowtie n\rangle \in \mathcal{A}$, then $\mathcal{E}(R, \langle \mathcal{V}(a), \mathcal{V}(b)\rangle) \bowtie n$.*

Proposition 3 *$\Sigma = \langle \mathcal{T}, \mathcal{A}\rangle$ is consistent iff there exists a fuzzy tableau for Σ.*

[8] A fuzzy $\mathcal{ALC}$ concept can be transformed into an equivalent one in NNF by pushing negations inwards using a combination of the De Morgan laws and the equivalences $\neg\exists R.C \equiv \forall R.\neg C$, $\neg\forall R.C \equiv \exists R.\neg C$.

Proof: [Sketch] For the *if* direction if $T = (\mathbf{S}, \mathcal{L}, \mathcal{E}, \mathcal{V})$ is a fuzzy tableau for Σ, we can construct a fuzzy interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ that is a model of $\mathcal{A}$ and $\mathcal{T}$ as follows:

$$\Delta^{\mathcal{I}} = \mathbf{S} \quad a^{\mathcal{I}} = \mathcal{V}(a), a \text{ occurs in } \mathcal{A} \quad A^{\mathcal{I}}(s) = \mathcal{L}(s, A) \text{ for all } s \in \mathbf{S}$$
$$\top^{\mathcal{I}}(s) = \mathcal{L}(s, \top), \bot^{\mathcal{I}}(s) = \mathcal{L}(s, \bot), \text{ for all } s \in \mathbf{S}$$
$$R^{\mathcal{I}}(s,t) = \mathcal{E}(R, \langle s, t\rangle) \text{ for all } \langle s, t\rangle \in \mathbf{S} \times \mathbf{S}$$

To prove that $\mathcal{I}$ is a model of $\mathcal{A}$ and $\mathcal{T}$, we can show by induction on the structure of concepts that $\mathcal{L}(s, C) \bowtie n$ implies $C^{\mathcal{I}}(s) \bowtie n$ for all $s \in \mathbf{S}$. Together with properties 12, 13, Proposition 1, the fact that we can restrict our attention to the degrees in $N^{\mathcal{A}}$ and the interpretation of individuals and roles, this implies that $\mathcal{I}$ is a model of $\mathcal{T}$, and that it satisfies each fuzzy assertion in $\mathcal{A}$.

For the converse, if $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ is a model of Σ, then a fuzzy tableau $T = (\mathbf{S}, \mathcal{L}, \mathcal{E}, \mathcal{V})$ for Σ can be defined as follows:

$$\mathbf{S} = \Delta^{\mathcal{I}} \quad \mathcal{E}(R, \langle s, t\rangle) = R^{\mathcal{I}}(s,t) \quad \mathcal{L}(s, C) = C^{\mathcal{I}}(s) \quad \mathcal{V}(a) = a^{\mathcal{I}}$$

It is easy to show that T is a fuzzy tableau for Σ. □

3.2 An algorithm for constructing a fuzzy tableau

In order to decide the consistency of $\Sigma = \langle \mathcal{T}, \mathcal{A}\rangle$ a procedure that constructs a fuzzy tableau T for Σ has to be determined. Like the tableaux algorithm presented in [8], our algorithm works on *completion-forests* since an ABox might contain several individuals with arbitrary roles connecting them. Due to the presence of general or cyclic terminologies, the termination of the algorithm is ensured by the use of *blocking*, where an expansion is terminated when individuals on the same path are asserted to belong to the same concepts.

Definition 2 *Let $\Sigma = \langle \mathcal{T}, \mathcal{A}\rangle$ be a fuzzy KB. A completion-forest $\mathcal{F}$ for Σ is a collection of trees whose distinguished roots are arbitrarily connected by edges. Each node x is labelled with a set $\mathcal{L}(x) = \{\langle C, \bowtie, n\rangle\}$, where $C \in sub(\Sigma)$, $\bowtie \in \{\geq, \leq\}$ and $n \in [-\epsilon, 1+\epsilon]$. Each edge $\langle x, y\rangle$ is labelled with a set $\mathcal{L}(\langle x, y\rangle) = \{\langle R, \bowtie, n\rangle\}$, where $R \in \boldsymbol{R}_\Sigma$ are roles occurring in Σ. Two triples $\langle C, \geq, n\rangle$ ($\langle R, \geq, n\rangle$) and $\langle C, \leq, m\rangle$ ($\langle R, \leq, m\rangle$) are* conjugated *if $n > m$.*

If nodes x and y are connected by an edge $\langle x, y\rangle$ with $\langle R, \bowtie, n\rangle \in \mathcal{L}(\langle x, y\rangle)$, then y is called an $R_{\bowtie n}$-successor *of x and x is called an $R_{\bowtie n}$*-predecessor *of y. Let y be an $R_{\geq n}$-successor of x, the edge $\langle x, y\rangle$ is* conjugated *with triples $\langle R, \leq, m\rangle$ if $n > m$. Similarly, we can extend it to the cases of $R_{\leq n}$-successor. A node x is an R-successor (resp. R-predecessor) of y if it is an $R_{\bowtie n}$-successor (resp. $R_{\bowtie n}$-predecessor) of y for some role R. As usual,* ancestor *is the transitive closure of* predecessor.

A node x is directly blocked *iff none of its ancestors are blocked, it is not a root node, and it has an ancestor y such that $\mathcal{L}(x) \subseteq \mathcal{L}(y)$. In this case, we say y directly blocks x. A node x is* blocked *iff it is directly blocked or if one of its predecessor is blocked.*

A node x is said to contain a clash *iff there exist two conjugated triples in $\mathcal{L}(x)$ or one of the following triples exists within $\mathcal{L}(x)$: (i) $\langle \bot, \geq, n\rangle$, for $n > 0$; (ii) $\langle \top, \leq, n\rangle$, for $n < 1$; (iii) $\langle C, \leq, -\epsilon\rangle$; (iv) $\langle C, \geq, 1+\epsilon\rangle$. The notion of '$\mathcal{L}(\langle x, y\rangle)$ contains a clash' is defined similarly.*

The algorithm initializes a forest $\mathcal{F}$ to contain (i) a root node x_0^i, for each individual a_i occurring in $\mathcal{A}$, labelled with $\mathcal{L}(x_0^i)$ such that $\{\langle C_i, \bowtie, n\rangle\} \subseteq \mathcal{L}(x_0^i)$ for each fuzzy assertion $\langle a_i{:}C_i \bowtie n\rangle \in \mathcal{A}$, and (ii) an edge $\langle x_0^i, x_0^j\rangle$, for each fuzzy assertion $\langle (a_i, a_j){:}R_i \bowtie n\rangle \in \mathcal{A}$, labelled with $\mathcal{L}(\langle x_0^i, x_0^j\rangle)$ such that $\{\langle R_i, \bowtie, n\rangle\} \subseteq \mathcal{L}(\langle x_0^i, x_0^j\rangle)$. $\mathcal{F}$ is then expanded by repeatedly

Table 2. Tableaux expansion rules

Rule		Description
$(\neg)$	if 1.	$\langle \neg C, \bowtie, n\rangle \in \mathcal{L}(x)$, and
	2.	$\langle C, \bowtie^-, 1-n\rangle \notin \mathcal{L}(x)$
	then	$\mathcal{L}(x) \to \mathcal{L}(x) \cup \{\langle C, \bowtie^-, 1-n\rangle\}$
$(\sqcap_\geq)$	if 1.	$\langle C_1 \sqcap C_2, \geq, n\rangle \in \mathcal{L}(x)$ and
	2.	$\{\langle C_1, \geq, n\rangle, \langle C_2, \geq, n\rangle\} \not\subseteq \mathcal{L}(x)$
	then	$\mathcal{L}(x) \to \mathcal{L}(x) \cup \{\langle C_1, \geq, n\rangle, \langle C_2, \geq, n\rangle\}$
$(\sqcup_\leq)$	if 1.	$\langle C_1 \sqcup C_2, \leq, n\rangle \in \mathcal{L}(x)$ and
	2.	$\{\langle C_1, \leq, n\rangle, \langle C_2, \leq, n\rangle\} \not\subseteq \mathcal{L}(x)$
	then	$\mathcal{L}(x) \to \mathcal{L}(x) \cup \{\langle C_1, \leq, n\rangle, \langle C_2, \leq, n\rangle\}$
$(\sqcup_\geq)$	if 1.	$\langle C_1 \sqcup C_2, \geq, n\rangle \in \mathcal{L}(x)$ and
	2.	$\{\langle C_1, \geq, n\rangle, \langle C_2, \geq, n\rangle\} \cap \mathcal{L}(x) = \emptyset$
	then	$\mathcal{L}(x) \to \mathcal{L}(x) \cup \{C\}$ for some
		$C \in \{\langle C_1, \geq, n\rangle, \langle C_2, \geq, n\rangle\}$
$(\sqcap_\leq)$	if 1.	$\langle C_1 \sqcap C_2, \leq, n\rangle \mathcal{L}(x)$ and
	2.	$\{\langle C_1, \leq, n\rangle, \langle C_2, \leq, n\rangle\} \cap \mathcal{L}(x) = \emptyset$
	then	$\mathcal{L}(x) \to \mathcal{L}(x) \cup \{C\}$ for some
		$C \in \{\langle C_1, \leq, n\rangle, \langle C_2, \leq, n\rangle\}$

Rule		Description
$(\exists_\geq)$	if 1.	$\langle \exists R.C, \geq, n\rangle \in \mathcal{L}(x)$, x is not blocked,
	2.	x has no $R_{\geq n}$-successor y with $\langle C, \geq, n\rangle \in \mathcal{L}(y)$
	then	create a new node y with $\mathcal{L}(\langle x, y\rangle) = \{\langle R, \geq, n\rangle\}$,
		$\mathcal{L}(y) = \{\langle C, \geq, n\rangle\}$,
$(\forall_\leq)$	if 1.	$\langle \forall R.C, \leq, n\rangle \in \mathcal{L}(x)$, x is not blocked,
	2.	x has no $R_{\geq 1-n}$-successor y with $\langle C, \leq, n\rangle \in \mathcal{L}(y)$
	then	create a new node y with $\mathcal{L}(\langle x, y\rangle) = \{\langle R, \geq, 1-n\rangle\}$,
		$\mathcal{L}(y) = \{\langle C, \leq, n\rangle\}$,
$(\forall_\geq)$	if 1.	$\langle \forall R.C, \geq, n\rangle \in \mathcal{L}(x)$,
	2.	x has an R-successor y with $\langle C, \geq, n\rangle \notin \mathcal{L}(y)$ and
	3.	$\langle R, \leq, 1-n\rangle$ is conjugated with the edge $\langle x, y\rangle$
	then	$\mathcal{L}(y) \to \mathcal{L}(y) \cup \{\langle C, \geq, n\rangle\}$,
$(\exists_\leq)$	if 1.	$\langle \exists R.C, \leq, n\rangle \in \mathcal{L}(x)$,
	2.	x has an R-successor y with $\langle C, \leq, n\rangle \notin \mathcal{L}(y)$ and
	3.	$\langle R, \leq, n\rangle$ is conjugated with the edge $\langle x, y\rangle$
	then	$\mathcal{L}(y) \to \mathcal{L}(y) \cup \{\langle C, \leq, n\rangle\}$,
$(\sqsubseteq)$	if 1.	$C \sqsubseteq D \in \mathcal{T}$ and
	2.	$\{\langle C, \leq, n-\epsilon\rangle, \langle D, \geq, n\rangle\} \cap \mathcal{L}(x) = \emptyset$ for $n \in N^{\mathcal{A}}$
	then	$\mathcal{L}(x) \to \mathcal{L}(x) \cup \{E\}$ for some
		$E \in \{\langle C, \leq, n-\epsilon\rangle, \langle D, \geq, n\rangle\}$

applying the completion rules from Table 2. The completion forest is complete when, for some node x (edge $\langle x, y\rangle$), $\mathcal{L}(x)$ ($\mathcal{L}(\langle x, y\rangle)$) contains a clash, or none of the completion rules in Table 2 are applicable. The algorithm stops when a clash occurs; it answers 'Σ is consistent' iff the completion rules can be applied in such a way that they yield a complete and clash-free completion forest, and 'Σ is inconsistent' otherwise.

From Table 2, we can see that for an arbitrary fuzzy assertion of the form $\langle a{:}D \bowtie n\rangle$ either value n or its complement $1-n$ appear in the expansion of a node x where $D \in \mathcal{L}(x)$. The finite property of the membership degrees makes blocking possible in our algorithm.

Example 1 *Let us show how the blocking condition works on the cyclic fuzzy KB, $\Sigma = \langle\{\texttt{HotPinkRose} \sqsubseteq \exists\texttt{nextGen}.\texttt{HotPinkRose}\}, \{\langle a{:}\texttt{HotPinkRose} \geq 0.6\rangle\}\rangle$. Σ is satisfiable and $N^{\mathcal{A}} = \{0, 0.5, 1\} \cup \{0.4, 0.6\}$. We start with a root node x^a, with label $\mathcal{L}(x^a) = \{\langle\texttt{HotPinkRose}, \geq, 0.6\rangle\}$. By applying rule $(\sqsubseteq)$ to node x^a, to $\texttt{HotPinkRose} \sqsubseteq \exists\texttt{nextGen}.\texttt{HotPinkRose}$ with, e.g., $n = 0.6$, and $E = \langle\exists\texttt{nextGen}.\texttt{HotPinkRose}, \geq, n\rangle$, we update the label $\mathcal{L}(x^a)$ with $\mathcal{L}(x^a) = \{\langle\texttt{HotPinkRose}, \geq, 0.6\rangle, \langle\exists\texttt{nextGen}.\texttt{HotPinkRose}, \geq, 0.6\rangle\}$. Continuing with node x^a, we apply rule $(\exists_\geq)$ to $\langle\exists\texttt{nextGen}.\texttt{HotPinkRose}, \geq, 0.6\rangle$, create a new edge $\langle x^a, y_1\rangle$ with $\mathcal{L}(\langle x^a, y_1\rangle) = \{\langle\texttt{nextGen}, \geq, 0.6\rangle\}$ and $\mathcal{L}(y_1) = \{\langle\texttt{HotPinkRose}, \geq, 0.6\rangle\}$. By continuing with node y_1 exactly as for node x_a, after applying rule $(\sqsubseteq)$, we update the label $\mathcal{L}(y_1)$ with $\mathcal{L}(y_1) = \{\langle\texttt{HotPinkRose}, \geq, 0.6\rangle, \langle\exists\texttt{nextGen}.\texttt{HotPinkRose}, \geq, 0.6\rangle\}$. Now, y_1 is an $\texttt{nextGen}$-successor of x^a and $\mathcal{L}(y_1) = \mathcal{L}(x^a)$ and, thus, y_1 is directly blocked.*

Example 2 *We show that $\Sigma = \langle\{C \sqsubseteq D\}, \{\langle a{:}C > 0.3\rangle, \langle a{:}D \leq 0.3\rangle\}\rangle$ is inconsistent. We first normalize Σ into $\Sigma = \langle\{C \sqsubseteq D\}, \{\langle a{:}C \geq 0.3+\epsilon\rangle, \langle a{:}D \leq 0.3\rangle\}\rangle$, for a small $\epsilon > 0$, e.g. $\epsilon = 0.01$. $N^{\mathcal{A}} = \{0, 0.5, 1\} \cup \{0.3, 0.3+\epsilon, 0.7-\epsilon, 0.7\}$. We start with a root node x^a and $\mathcal{L}(x^a) = \{\langle C, \geq, 0.3+\epsilon\rangle, \langle D, \leq, 0.3\rangle\}$. By applying rule $(\sqsubseteq)$ to node x^a, to $C \sqsubseteq D$ with, e.g., $n = 0.3+\epsilon$ we get two branches, depending on the 'choice of $E \in \{\langle C, \leq, 0.3+\epsilon-\epsilon\rangle, \langle D, \geq, 0.3+\epsilon\rangle\}$'. In the former case, we update the label $\mathcal{L}(x^a)$ with $\mathcal{L}(x^a) = \{\langle C, \geq, 0.3+\epsilon\rangle, \langle D, \leq, 0.3\rangle, \langle C, \leq, 0.3\rangle\}$ which contains a clash, while in the latter case we update the label $\mathcal{L}(x^a)$ with $\mathcal{L}(x^a) = \{\langle C, \geq, 0.3+\epsilon\rangle, \langle D, \leq, 0.3\rangle, \langle D, \geq, 0.3+\epsilon\rangle\}$ which also contains a clash. No complete, clash-free forest can be obtained, thus the algorithm answers with 'inconsistent'.*

Proposition 4 (Termination) *For each fuzzy $\mathcal{ALC}$ KB Σ, the tableau algorithm terminates.*

Proof: [Sketch] Termination is a result of the properties of the expansion rules, as in the classical case [8]. More precisely we have the following observations. (i) The expansion rules never remove nodes from the tree or concepts from node labels or change the edge labels. (ii) Successors are only generated by the rules $\exists_\geq$ and $\forall_\leq$. For any node and for each concept these rules are applied at-most once. (iii) Since nodes are labelled with nonempty subsets of $sub(\Sigma)$, obviously there is a finite number of possible labellings for a pair of nodes. Thus, the condition of blocking will be applied in any path of the tree and consequently any path will have a finite length. $\Box$

Proposition 5 (Soundness) *If the expansion rules can be applied to an fuzzy $\mathcal{ALC}$ KB Σ such that they yield a complete and clash-free completion-forest, then Σ has a fuzzy tableau for Σ.*

Proof: [Sketch] Let $\mathcal{F}$ be a complete and clash-free completion-forest constructed by the tableaux algorithm for Σ. A fuzzy tableau $T = (\mathbf{S}, \mathcal{L}, \mathcal{E}, \mathcal{V})$ can be defined as follows:

$$
\begin{array}{rcl}
\mathbf{S} & = & \{x \mid x \text{ is a node in } \mathcal{F}, \text{ and } x \text{ is not blocked}\}, \\
\mathcal{L}(x, \bot) & = & 0, \text{ for all } x \in \mathbf{S}, \\
\mathcal{L}(x, \top) & = & 1, \text{ for all } x \in \mathbf{S}, \\
\mathcal{L}(x, C) & = & \max[\langle C, \geq, n_i\rangle], \text{ for all } x \text{ in } \mathcal{F} \text{ not blocked}, \\
\mathcal{L}(x, \neg A) & = & 1 - \mathcal{L}(x, A), \text{ for all } x \text{ in } \mathcal{F} \text{ not blocked}, \\
 & & \text{with } \langle \neg A, \geq, n\rangle \in \mathcal{L}(x), \\
\mathcal{E}(R, \langle x, y\rangle) & = & \{\max[\langle R, \geq, n_i\rangle] \mid \\
 & & \quad 1.\ y \text{ is an } R_{\geq n_i}\text{-successor of } x \textit{ or} \\
 & & \quad 2. \langle R, \geq, n_i\rangle \in \mathcal{L}(\langle x, z\rangle) \text{ and } y \text{ blocks } z\}, \\
\mathcal{V}(a_i) & = & x_0^i, \text{ where } x_0^i \text{ is a root node}
\end{array}
$$

where max returns the maximum degree n out of the set of triples of the form $\langle A, \geq, n_i\rangle$, or 0 if no such triple exists. It can be shown that T is a fuzzy tableau for Σ. $\Box$

Proposition 6 (Completeness) *Consider a fuzzy $\mathcal{ALC}$ KB Σ. If Σ has a fuzzy tableau, then the expansion rules can be applied in such a way that the tableaux algorithm yields a complete and clash-free completion-forest for Σ.*

Proof: [Sketch] Let $T = (\mathbf{S}, \mathcal{L}, \mathcal{E}, \mathcal{V})$ be a fuzzy tableau for Σ. Using T, we trigger the application of the expansion rules such that they yield a completion-forest $\mathcal{F}$ that is both complete and clash-free. Similarly to [8] we can define a mapping π which maps nodes of $\mathcal{F}$ to elements of $\mathbf{S}$, and guide the application of the non-deterministic rules $\sqsubseteq$, $\sqcup_{\geq}$ and $\sqcap_{\leq}$. Our method slightly differs from the one used in crisp DLs [8] in the following way. Using the membership degree of a node to a concept, found in the fuzzy tableau, we create artificial triples that are tested against conjugation with the candidate triples that the non-deterministic rule can insert in the completion-forest. The triples that don't cause a conjugation can be added. The modified rules, which are used to guide such an expansion, are presented in Table 3. The modified rules together with the termination property ensure the completeness of the algorithm. $\square$

Table 3. The $\sqsubseteq'$, $\sqcup'_{\geq}$ and $\sqcap'_{\leq}$ rules

($\sqsubseteq'$)	if 1.	$C \sqsubseteq D \in \mathcal{T}$, x is not indirectly blocked and
	2.	$\{\langle C, \leq, n-\epsilon\rangle, \langle D, \geq, n\rangle\} \cap \mathcal{L}(x) = \emptyset$ for $n \in N^{\mathcal{A}}$
	then	$\mathcal{L}(x) \rightarrow \mathcal{L}(x) \cup \{E\}$ for some $E \in \{\langle C, \leq, n-\epsilon\rangle$, $\langle D, \geq, n\rangle\}$, not conjugated with $\langle C, \geq, \mathcal{L}(\pi(x), C)\rangle$ or $\langle D, \leq, \mathcal{L}(\pi(x), D)\rangle$
($\sqcup'_{\geq}$)	if 1.	$\langle C_1 \sqcup C_2, \geq, n\rangle \in \mathcal{L}(x)$, x is not indirectly blocked, and
	2.	$\{\langle C_1, \geq, n\rangle, \langle C_2, \geq, n\rangle\} \cap \mathcal{L}(x) = \emptyset$
	then	$\mathcal{L}(x) \rightarrow \mathcal{L}(x) \cup \{C\}$ for some $C \in \{\langle C_1, \geq, n\rangle$, $\langle C_2, \geq, n\rangle\}$ not conjugated with $\langle C_1, \leq, \mathcal{L}(\pi(x), C_1)\rangle$ or $\langle C_2, \leq, \mathcal{L}(\pi(x), C_2)\rangle$
($\sqcap'_{\leq}$)	if 1.	$\langle C_1 \sqcap C_2, \leq, n\rangle \in \mathcal{L}(x)$, x is not indirectly blocked, and
	2.	$\{\langle C_1, \leq, n\rangle, \langle C_2, \leq, n\rangle\} \cap \mathcal{L}(x) = \emptyset$
	then	$\mathcal{L}(x) \rightarrow \mathcal{L}(x) \cup \{C\}$ for some $C \in \{\langle C_1, \leq, n\rangle$, $\langle C_2, \leq, n\rangle\}$ not conjugated with $\langle C_1, \geq, \mathcal{L}(\pi(x), C_1)\rangle$ or $\langle C_2, \geq, \mathcal{L}(\pi(x), C_2)\rangle$

4 CONCLUSIONS

Fuzzy DLs extend crisp DLs to deal with vague concepts. None of the work on fuzzy DLs so far presented a correct and complete calculus for cyclic TBoxes and general concept inclusions, which are important features of current crisp DL systems. We overcome to this limitation by providing a tableau for fuzzy $\mathcal{ALC}$ with GCIs.

Major topics for future research are indeed the extension of the fuzzy tableau algorithm to expressive DL languages such as fuzzy $\mathcal{SHIF}(\mathtt{D})$ or $\mathcal{SHOIN}(\mathtt{D})$ [21] and the development of a system supporting this language. In the former case, such algorithm can be based directly on the ones presented for the fuzzy $\mathcal{SI}$ and $\mathcal{SHIN}$ DLs [14, 15] and the rules for nominals, for $\mathcal{SHOIN}$ [10] and for fuzzy $\mathcal{SHOIN}$ [16].

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