Towards Spatial Reasoning in Fuzzy Description Logics

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Introduction

- In the last years the interest in ontologies has significantly grown
- An ontology is defined as an explicit and formal specification of a shared conceptualization
- Description Logics (DLs) are a family of logics that are behind the standard ontology language OWL [HPS04].

- It is widely agreed that “classical” ontology languages are not appropriate to deal with *fuzzy/vague knowledge*
- Fuzzy ontologies emerge as useful in several applications, such as multimedia information retrieval, image interpretation, ontology mapping, matchmaking and the Semantic Web [LS08]
- Several fuzzy extensions of DLs can be found in the literature (see the survey in [LS08])
- Some fuzzy DL reasoners have been implemented, such as FUZZYDL [BS08], DELOREAN [BDGR08] or FIRE [SSSK06].

- In this work, we make a first step in extending fuzzy DLs towards fuzzy spatial reasoning by supporting both
  - Fuzzy spatial relations of the Region Connection Calculus (RCC) [EDH97, RZC92, SCK09]
  - Application domain dependent fuzzy spatial relations such as “close”, “far”, “over”
- Thus, offer a framework for modeling spatial relations such as

  “region a is part of region b, which is connected to region c, a is close to c and b is right over c”.

Preliminaries: Mathematical Fuzzy Logic [Háj98]

- Fuzzy statements: $\langle \phi, n \rangle$, where $n \in [0, 1]$ and ϕ is a statement
  - The degree of truth of ϕ is *at least n*
- Fuzzy interpretation: $\mathcal{I} : Atoms \to [0, 1]$ and is then extended inductively:

$$
\begin{array}{ll}
\mathcal{I}(\phi \wedge \psi) = \mathcal{I}(\phi) \otimes \mathcal{I}(\psi) & \mathcal{I}(\phi \vee \psi) = \mathcal{I}(\phi) \oplus \mathcal{I}(\psi), \\
\mathcal{I}(\phi \to \psi) = \mathcal{I}(\phi) \Rightarrow \mathcal{I}(\psi) & \mathcal{I}(\neg \phi) = \ominus \mathcal{I}(\phi), \\
\mathcal{I}(\exists x.\phi(x)) = \sup_{c \in \Delta^{\mathcal{I}}} \mathcal{I}(\phi(c)) & \mathcal{I}(\forall x.\phi(x)) = \inf_{c \in \Delta^{\mathcal{I}}} \mathcal{I}(\phi(c))
\end{array}
$$

$\otimes$, $\oplus$, $\Rightarrow$, and $\ominus$ are *truth combination functions*

	Łukasiewicz Logic	Gödel Logic	Product Logic	"Zadeh Logic"
$a \otimes b$	$\max(a + b - 1, 0)$	$\min(a, b)$	$a \cdot b$	$\min(a, b)$
$a \oplus b$	$\min(a + b, 1)$	$\max(a, b)$	$a + b - a \cdot b$	$\max(a, b)$
$a \Rightarrow b$	$\min(1 - a + b, 1)$	$\begin{cases} 1 & \text{if } a \leqslant b \\ b & \text{otherwise} \end{cases}$	$\min(1, b/a)$	$\max(1 - a, b)$
$\ominus a$	$1 - a$	$\begin{cases} 1 & \text{if } a = 0 \\ 0 & \text{otherwise} \end{cases}$	$\begin{cases} 1 & \text{if } a = 0 \\ 0 & \text{otherwise} \end{cases}$	$1 - a$

- $\mathcal{I} \models \langle \phi, n \rangle$ iff $\mathcal{I}(\phi) \geqslant n$
- Best Entailment Degree (BED):
  $bed(\mathcal{K}, \phi) = \sup \{r \mid \mathcal{K} \models \langle \phi, r \rangle\}$
- BED can be computed as (where $\phi \leqslant x$ is $\langle \neg\phi, 1 - x \rangle$)

$$bed(\mathcal{K}, \phi) = \min x. \text{ such that } \mathcal{K} \cup \{\phi \leqslant x\} \text{ satisfiable}$$

- E.g., for Łukasiewicz logic, we may use Mixed Integer Linear Programming

$$\begin{aligned} bed(\mathcal{K}, \phi) = \min x. \text{ such that } & \\ & x \in [0,1], x_{\neg\phi} \geqslant 1 - x, \sigma(\neg\phi), \\ & \text{for all } \langle \phi', n \rangle \in \mathcal{K}, x_{\phi'} \geqslant n, \sigma(\phi'), \end{aligned}$$

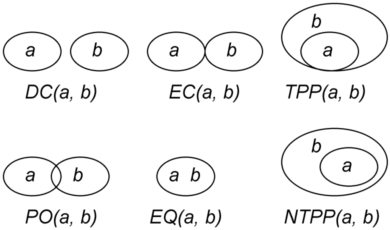
$$\sigma(\phi) = \begin{cases} x_p \in [0,1] & \text{if} \quad \phi = p \\ x_{\phi'} = \ominus x_\phi, x_\phi \in [0,1] & \text{if} \quad \phi = \neg\phi' \\ \begin{array}{l} x_{\phi_1} \otimes x_{\phi_2} = x_\phi, \\ \sigma(\phi_1), \sigma(\phi_2), x_\phi \in [0,1] \end{array} & \text{if} \quad \phi = \phi_1 \wedge \phi_2 \\ x_{\phi_1} \oplus x_{\phi_2} = x_\phi & \text{if} \quad \phi = \phi_1 \vee \phi_2 \\ \sigma(\neg\phi_1 \vee \phi_2) & \text{if} \quad \phi = \phi_1 \rightarrow \phi_2 \,. \end{cases}$$

- Based on Fuzzy Region Connection Calculus (f-RCC)
- We consider a fixed finite non-empty set of regions $\mathcal{R}$
- Topological relations: defined in terms of an arbitrary reflexive and symmetric relation C on $\mathcal{R}$, called connection
- $C(a, b)$ understood to mean that a is connected with b
- C is a fuzzy relation
  - for each pair $\langle a, b \rangle$ of regions, $\mathrm{C}(a, b)$ is a degree in $[0, 1]$ reflecting to what extent a and b are connected

- The axioms defining the topological relations for regions *a* and *b* (quantifiers range over $\mathcal{R}$)

Name	Relation	RCC definition
Disconnected	DC	$\neg C(a, b)$
Part	P	$\forall c.(C(c, a) \rightarrow C(c, b))$
Proper Part	PP	$P(a, b) \wedge \neg P(b, a)$
Equals	EQ	$P(a, b) \wedge P(b, a)$
Overlaps	O	$\exists c.(P(c, a) \wedge P(c, b))$
Discrete	DR	$\neg O(a, b)$
Partially Overlaps	PO	$O(a, b) \wedge \neg P(a, b) \wedge \neg P(b, a)$
Externally connected	EC	$C(a, b) \wedge \neg O(a, b)$
Non Tangential Part	NTP	$\forall c.(C(c, a) \rightarrow O(c, b))$
Tangential PP	TPP	$PP(a, b) \wedge \neg NTP(a, b)$
Non-Tangential PP	NTPP	$PP(a, b) \wedge NTP(a, b)$

- The intuitive meaning of some of these relations is



- A fuzzy spatial statement is of the form $\langle \phi, n \rangle$, where $n \in [0, 1]$ and ϕ is a boolean combination of RCC topological relations of the form $\mathsf{S}(a, b)$ with a, b regions and S topological relation
- A fuzzy spatial interpretation $\mathcal{I}$ maps each basic RCC statement $\mathsf{S}(a, b)$ into $[0, 1]$ according to the axiom for S

$$
\begin{aligned}
\mathcal{I}(\mathsf{P}(a,b) \rightarrow \mathsf{C}(b,d)) &= \mathcal{I}(\mathsf{P}(a,b)) \Rightarrow \mathcal{I}(\mathsf{C}(b,d)) \\
&= \mathcal{I}(\forall c.(\mathsf{C}(c,a) \rightarrow \mathsf{C}(c,b))) \Rightarrow \mathcal{I}(\mathsf{C}(b,c)) \\
&= \min_{c \in \mathcal{R}}\{(\mathcal{I}(\mathsf{C}(c,a)) \Rightarrow \mathcal{I}(\mathsf{C}(c,b))) \Rightarrow \mathcal{I}(\mathsf{C}(b,c))\} \, .
\end{aligned}
$$

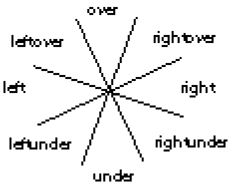
- Reasoning: similarly as before, except that we have to take into account that some RCC relations involve quantifiers
- Quantifiers are not a problem as the set of regions is considered finite
- It suffices to consider

$$\rho(\phi) = \begin{cases} \mathsf{C}(a,b) & \text{if} \quad \phi = \mathsf{C}(a,b) \\ \rho(\phi_{\mathsf{S}(a,b)}) & \text{if} \quad \phi = \mathsf{S}(a,b) \\ \neg\rho(\phi') & \text{if} \quad \phi = \neg\phi' \\ \rho(\phi') \wedge \rho(\phi'') & \text{if} \quad \phi = \phi' \wedge \phi'' \\ \rho(\phi') \vee \rho(\phi'') & \text{if} \quad \phi = \phi' \vee \phi'' \\ \rho(\phi') \rightarrow \rho(\phi'') & \text{if} \quad \phi = \phi' \rightarrow \phi'' \\ \bigvee^{G}_{c \in \mathcal{R}} \rho(\phi'(c)) & \text{if} \quad \phi = \exists x.\phi'(x) \\ \bigwedge^{G}_{c \in \mathcal{R}} \rho(\phi'(c)) & \text{if} \quad \phi = \forall x.\phi' \end{cases}$$

where

- $\bigwedge^{G}$ and $\bigvee^{G}$ are Gödel conjunction and disjunction, respectively
- $\phi_{\mathsf{S}(a,b)}$ is the definition of relation $\mathsf{S}(a,b)$

- We further extend the language
- So far, the spatial relations are those of RCC
- In many cases, we would also like to support other non RCC fuzzy spatial relations, such as angle-based spatial relations [SRF07, HAB08]



- The definition of such spatial relations S on regions is application dependent
  - Some fixed fuzzy membership function $\mu_{\mathsf{S}} \colon \mathcal{R} \times \mathcal{R} \to [0, 1]$ computing the degree of truth of $\mathsf{S}(a, b)$ is given
- We extend fuzzy spatial statements, by allowing domain dependent spatial expressions to occur
- For instance

$$\langle \mathsf{P}(a, b) \wedge \mathsf{C}(b, c) \wedge \mathsf{Close}(a, c) \wedge \mathsf{RightOver}(b, c), 0.8 \rangle$$

is a fuzzy spatial statement, with intended meaning “region *a* is part of region *b*, which is connected to region *c*, *a* is close to *c* and *b* is right over *c*”

Towards a Spatial Fuzzy Description Logic

- Our spatial fuzzy DL is grounded on the fuzzy DL $\mathcal{ALCF}(D)$ [Str05]
- We will just provide a minimal variant of $\mathcal{ALCF}(D)$ to deal with spatial reasoning
- Recall that $\mathcal{ALCF}(D)$ is the basic DL $\mathcal{ALC}$ extended with functional roles (letter $\mathcal{F}$) and concrete domains [LM07] (letter D) allowing to deal with data types such as strings, integers and reals
- In our specific fuzzy $\mathcal{ALCF}(D)$, the concrete domain consists of the set of regions $\mathcal{R}$ and spatial relations over it

Description Logics (DLs)

- The logics behind OWL-DL and OWL-Lite, `http://dl.kr.org/`.
- Concept/Class: names are equivalent to unary predicates
  - In general, concepts equiv to formulae with one free variable
- Role or attribute: names are equivalent to binary predicates
  - In general, roles equiv to formulae with two free variables
- Taxonomy: Concept and role hierarchies can be expressed
- Individual: names are equivalent to constants
- Operators: restricted so that:
  - Language is decidable and, if possible, of low complexity
  - No need for explicit use of variables
    - Restricted form of $\exists$ and $\forall$
  - Features such as counting can be succinctly expressed

The Crisp DL Family

- A given DL is defined by set of concept and role forming operators
- Basic language: $\mathcal{ALC}$($\mathcal{A}$ttributive $\mathcal{L}$anguage with $\mathcal{C}$omplement)

Syntax				Semantics	Example
C, D	$\rightarrow$	$\top$	|	$\top(x)$	
		$\bot$	|	$\bot(x)$	
		A	|	$A(x)$	$Human$
		$C \sqcap D$	|	$C(x) \wedge D(x)$	$Human \sqcap Male$
		$C \sqcup D$	|	$C(x) \vee D(x)$	$Nice \sqcup Rich$
		$\neg C$	|	$\neg C(x)$	$\neg Meat$
		$\exists R.C$	|	$\exists y.R(x,y) \wedge C(y)$	$\exists has\_child.Blond$
		$\forall R.C$		$\forall y.R(x,y) \rightarrow C(y)$	$\forall has\_child.Human$
$C \sqsubseteq D$				$\forall x.C(x) \rightarrow D(x)$	$Happy\_Father \sqsubseteq Man \sqcap \exists has\_child.Female$
$a{:}C$				$C(a)$	$John{:}Happy\_Father$

Toy Example

$$
\begin{array}{rcl}
Sex & = & Male \sqcup Female \\
Male \sqcap Female & \sqsubseteq & \bot \\
Person & \sqsubseteq & Human \sqcap \exists hasSex.Sex \\
MalePerson & \sqsubseteq & Person \sqcap \exists hasSex.Male
\end{array}
$$

$$umberto : Person \sqcap \exists hasSex.\neg Female$$

$$KB \models umberto : MalePerson$$

Note on DL Naming

$\mathcal{AL}$: $C, D \longrightarrow \top \mid \bot \mid A \mid C \sqcap D \mid \neg A \mid \exists R.\top \mid \forall R.C$

$\mathcal{C}$: Concept negation, $\neg C$. Thus, $\mathcal{ALC} = \mathcal{AL} + \mathcal{C}$

$\mathcal{S}$: Used for $\mathcal{ALC}$ with transitive roles $\mathcal{R}_+$

$\mathcal{U}$: Concept disjunction, $C_1 \sqcup C_2$

$\mathcal{E}$: Existential quantification, $\exists R.C$

$\mathcal{H}$: Role inclusion axioms, $R_1 \sqsubseteq R_2$, *e.g.*, *is_component_of* $\sqsubseteq$ *is_part_of*

$\mathcal{N}$: Number restrictions, $(\geqslant n\ R)$ and $(\leqslant n\ R)$, *e.g.*, $(\geqslant 3\ has\_Child)$ (has at least 3 children)

$\mathcal{Q}$: Qualified number restrictions, $(\geqslant n\ R.C)$ and $(\leqslant n\ R.C)$, *e.g.*, $(\leqslant 2\ has\_Child.Adult)$ (has at most 2 adult children)

$\mathcal{O}$: Nominals (singleton class), $\{a\}$, *e.g.*, $\exists has\_child.\{mary\}$.
Note: $a{:}C$ equiv to $\{a\} \sqsubseteq C$ and $(a, b){:}R$ equiv to $\{a\} \sqsubseteq \exists R.\{b\}$

$\mathcal{I}$: Inverse role, R^-, *e.g.*, $isPartOf = hasPart^-$

$\mathcal{F}$: Functional role, f, *e.g.*, $functional(hasAge)$

$\mathcal{R}_+$: transitive role, *e.g.*, $transitive(isPartOf)$

$\mathcal{R}$: role inclusions with composition, $R_1 \circ R_2 \sqsubseteq S$, *e.g.*, $isPartOf \circ isPartOf \sqsubseteq isPartOf$

For instance,

$$\mathcal{SHIF} = \mathcal{S} + \mathcal{H} + \mathcal{I} + \mathcal{F} = \mathcal{ALCR_+HIF} \quad \text{OWL-Lite}$$
$$\mathcal{SHOIN} = \mathcal{S} + \mathcal{H} + \mathcal{O} + \mathcal{I} + \mathcal{N} = \mathcal{ALCR_+HOIN} \quad \text{OWL-DL}$$
$$\mathcal{SROIQ} = \mathcal{S} + \mathcal{R} + \mathcal{O} + \mathcal{I} + \mathcal{Q} = \mathcal{ALCR_+ROIN} \quad \text{OWL 2}$$

Concrete Domains

- Concrete domains: reals, integers, strings, ...

  $(tim, 14) : hasAge$
  $(sf, \text{“}SoftComputing\text{”}) : hasAcronym$
  $(source1, \text{“}ComputerScience\text{”}) : isAbout$
  $(service2, \text{“}InformationRetrievalTool'') : Matches$
  $Minor = Person \sqcap \exists hasAge. \leqslant_{18}$

- Semantics: a clean separation between “object” classes and concrete domains
  - $D = \langle \Delta_D, \Phi_D \rangle$
  - Δ_D is an interpretation domain
  - Φ_D is the set of concrete domain predicates d with a predefined arity n and fixed interpretation $d^D \subseteq \Delta_D^n$
  - Concrete properties: $R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta_D$
- Notation: (D). E.g., $\mathcal{ALC}(D)$ is $\mathcal{ALC}$ + concrete domains

Fuzzy DLs Basics

The semantics is an immediate consequence of applying mathematical fuzzy logic to the First-Order-Logic translation of DLs expressions

Interpretation:

$$\mathcal{I} = \Delta^{\mathcal{I}} \qquad C^{\mathcal{I}} : \Delta^{\mathcal{I}} \to [0,1] \qquad R^{\mathcal{I}} : \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \to [0,1]$$

$\otimes$ = t-norm
$\oplus$ = s-norm
$\ominus$ = negation
$\Rightarrow$ = implication

Concepts:

Syntax		Semantics		
$C, D \longrightarrow$	$\top$	$\top^{\mathcal{I}}(x)$	$=$	1
	$\bot$	$\bot^{\mathcal{I}}(x)$	$=$	0
	A	$A^{\mathcal{I}}(x)$	$\in$	$[0,1]$
	$C \sqcap D$	$(C_1 \sqcap C_2)^{\mathcal{I}}(x)$	$=$	${C_1}^{\mathcal{I}}(x) \otimes {C_2}^{\mathcal{I}}(x)$
	$C \sqcup D$	$(C_1 \sqcup C_2)^{\mathcal{I}}(x)$	$=$	${C_1}^{\mathcal{I}}(x) \oplus {C_2}^{\mathcal{I}}(x)$
	$\neg C$	$(\neg C)^{\mathcal{I}}(x)$	$=$	$\ominus C^{\mathcal{I}}(x)$
	$\exists R.C$	$(\exists R.C)^{\mathcal{I}}(x)$	$=$	$\sup_{y \in \Delta^{\mathcal{I}}} R^{\mathcal{I}}(x,y) \otimes C^{\mathcal{I}}(y)$
	$\forall R.C$	$(\forall R.C)^{\mathcal{I}}(u)$	$=$	$\inf_{y \in \Delta^{\mathcal{I}}} R^{\mathcal{I}}(x,y) \Rightarrow C^{\mathcal{I}}(y)\}$

Assertions: $\langle a{:}C, n\rangle, \mathcal{I} \models \langle a{:}C, n\rangle$ iff $C^{\mathcal{I}}(a^{\mathcal{I}}) \geqslant n$ (similarly for roles)

- individual a is instance of concept C at least to degree n, $n \in [0,1] \cap \mathbb{Q}$

Inclusion axioms: $\langle C \sqsubseteq D, n\rangle$,

- $\mathcal{I} \models \langle C \sqsubseteq D, n\rangle$ iff $\inf_{x \in \Delta^{\mathcal{I}}} C^{\mathcal{I}}(x) \Rightarrow D^{\mathcal{I}}(x) \geqslant n$

Definition (Spatial Fuzzy Concept Expressions)

$$C \rightarrow \quad \forall(T, T').\mathbf{d} \quad | \quad \text{(concrete universal quantification)}$$
$$\exists(T, T').\mathbf{d} \quad | \quad \text{(concrete existential quantification)}$$

where **d** is a boolean combination of fuzzy domain predicate names, called spatial expression

For instance

$$\mathsf{House} \sqcap \exists(\mathsf{hasLoc}, \mathsf{hasCarParkLoc}).\mathsf{Close}$$

is a concept that informally determines the degree of being an object a house having a car park close to it

Semantics

Syntax				FOL	Example
C, C'	$\rightarrow$	$\top$	|	$\top(x)$	
		$\bot$	|	$\bot(x)$	
		A	|	$A(x)$	Human
		$C \sqcap C'$	|	$C(x) \wedge C'(x)$	Human $\sqcap$ Male
		$C \sqcup C'$	|	$C(x) \vee C'(x)$	Nice $\sqcup$ Rich
		$\neg C$	|	$\neg C(x)$	$\neg$Meat
		$\exists R.C$	|	$\exists y.R(x, y) \wedge C(y)$	$\exists$has_child.Blond
		$\forall R.C$		$\forall y.R(x, y) \rightarrow C(y)$	$\forall$has_child.Human
		$\exists(T, T').\mathbf{d}$	|	$\exists r \exists r'.T(x, r) \wedge T'(x, r') \wedge \mathbf{d}(r, r')$	$\exists$(has_region, has_region).Over
		$\forall(T, T').\mathbf{d}$		$\forall r \forall r'.(T(x, r) \wedge T'(x, r')) \rightarrow \mathbf{d}(r, r')$	$\forall$(has_region, has_region).LeftUnder

Example

- Assume we would like to recognize whether there is a quite house in an image (see, *e.g.*, [MSS01])
- A quite house is defined as a house having a garden around
- Assume that we have a definition for quite house and the output of a (semi-automatic) image classification tool applied to an image i:

$$\begin{aligned}\mathsf{QuiteHouseImage} = {} & \mathsf{Image} \sqcap \exists \mathsf{depicts.House} \sqcap \exists \mathsf{depicts.Garden} \\ & \sqcap \exists (\mathsf{hasHouseLocation}, \mathsf{hasGardenLocation}).(\mathsf{Around} \wedge \mathsf{EC})\end{aligned}$$

$$\begin{aligned}&\langle \mathsf{i} : \mathsf{Image}, 1\rangle, \langle \mathsf{i} : \exists \mathsf{depicts.House}, 0.8\rangle, \langle \mathsf{i} : \exists \mathsf{depicts.Garden}, 0.7\rangle, \\ &\langle (\mathsf{i}, \mathsf{r}) : \mathsf{hasHouseLocation}, 1\rangle, \langle (\mathsf{i}, \mathsf{r}') : \mathsf{hasGardenLocation}, 1\rangle, \\ &\langle \mathsf{i} : \exists (\mathsf{hasHouseLocation}, \mathsf{hasGardenLocation}).(\mathsf{C} \wedge \neg \mathsf{O}), 0.8\rangle, \\ &\langle (\mathsf{r}, \mathsf{r}') : \mathsf{Around}, 0.9\rangle \; .\end{aligned}$$

- It can be shown that (under Zadeh semantics) $\mathit{bed}(\mathcal{K}, \mathsf{i} : \mathsf{QuiteHouseImage}) = 0.8 \otimes 0.7 \otimes 0.8 \otimes 0.9 = 0.7$
- Thus, the image depicts a quite house to degree 0.7

Reasoning

- Combine reasoning method for fuzzy DLs with that for fuzzy spatial reasoning (see paper)

Conclusions & Outlook

- We have made a first attempt towards spatial reasoning in fuzzy Description Logics by adding
  - Fuzzy topological relations of the Region Connection Calculus
  - Application dependent spatial relations
- We are planning to include it into our reasoner FUZZYDL
- So far, we assume that a fixed finite set of regions is provided as input.
  - We would like to extend ou result to the case where this is not required

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