On Qualified Cardinality Restrictions in Fuzzy Description Logics under Łukasiewicz semantics

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IPMU 2008

Torremolinos, Málaga (Spain), June 2008

Outline

1. Introduction
2. Fuzzy Logic
3. The Fuzzy DL $\mathcal{ALCQ}$
4. Reasoning algorithm
5. Conclusions and future work

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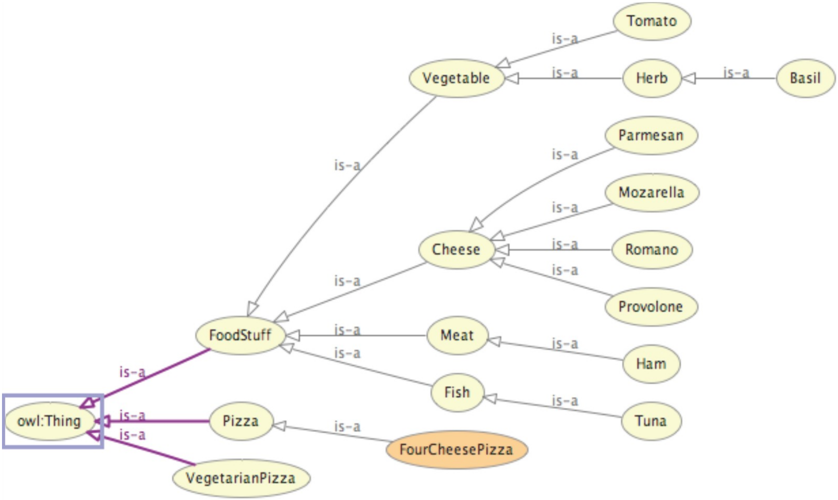
Description Logics

- **Description Logics** (DLs) are a family of logics for representing structured knowledge.
- They describe a domain in terms of concepts (classes), roles (properties, relationships) and individuals.
  - Complex concepts and roles can be built.
- Distinguished by a formal semantics and by providing some inference services: consistency, subsumption ...
- More popularity due to their application in **Semantic Web**, as the theoretical counterpart of the standard language for ontology representation OWL (Web Ontology Language).
  - OWL DL is the highest level such that reasoning is decidable.
- Each logic is denoted by using a string of capital letters which identify the constructs of the logic and therefore its complexity.
  - For instance, $\mathcal{ALCQ}$ is a subset of $\mathcal{SHOIN}(\mathbf{D})$, which is the subjacent logic of OWL DL language.

Ontology in a Description-Logic Based Language



Fuzzy Description Logics

- DLs are not appropriate for **fuzzy/vague/imprecise** knowledge for which a clear and precise definition is not possible.
  - An inn is a cheap and small hotel.
  - Patient001's Serotonin Level is quite low.
  - English is generally spoken in Canada.
  - I do not like flamenco very much.
- **Fuzzy logic** and fuzzy set theory are well known formalisms for representing these types of knowledge.
- **Fuzzy DLs** extend DLs with fuzzy logic.

Our contributions

- Relative little work has been done in reasoning in fuzzy DLs with **qualified cardinality restrictions**.
- They are an important feature on DLs. For instance, they allow to define a father having two daughters as:

$$Man \sqcap (\geqslant \mathbf{2}\ hasChild.Woman)$$

- In fact, they are one of the main motivations for extending the current standard language OWL to OWL 1.1.
- In this work we present a fuzzy extension of $\mathcal{ALCQ}$, with qualified cardinality restrictions, under **Łukasiewicz** semantics.
- We will analyze the behaviour of the constructor, propose a new semantics and provide a reasoning algorithm.

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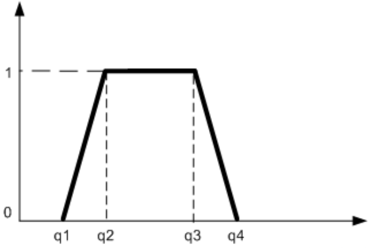
Fuzzy logic

- Fuzzy set theory and fuzzy logic (Zadeh, 1965) manage imprecise and vague knowledge.
- In classical set theory elements either belong to a set or not.
- In fuzzy set theory elements can belong to **some degree in** $[0, 1]$.
  - 0 means no-membership.
  - 1 means full membership.
  - A value between 0 and 1 represents the extent to which an element can be considered as an element of the fuzzy set.
- Example: **Trapezoidal** membership function of a fuzzy set:



Families of fuzzy logics

- All crisp set **operations** are extended to fuzzy sets.
  - Intersection: t-norm function.
  - Union: t-conorm function.
  - Complement: negation function.
  - Implication: implication function.
- Fuzzy operators are grouped in **families**

Family	t-norm $\alpha \otimes \beta$	t-conorm $\alpha \oplus \beta$	negation $\ominus \alpha$	implication $\alpha \Rightarrow \beta$
Zadeh	$\min\{\alpha, \beta\}$	$\max\{\alpha, \beta\}$	$1 - \alpha$	$\max\{1 - \alpha, \beta\}$
Łukasiewicz	$\max\{\alpha + \beta - 1, 0\}$	$\min\{\alpha + \beta, 1\}$	$1 - \alpha$	$\min\{1 - \alpha + \beta, 1\}$
Gödel	$\min\{\alpha, \beta\}$	$\max\{\alpha, \beta\}$	$\begin{cases} 1, & \alpha = 0 \\ 0, & \alpha > 0 \end{cases}$	$\begin{cases} 1 & \alpha \leqslant \beta \\ \beta, & \alpha > \beta \end{cases}$
Product	$\alpha \cdot \beta$	$\alpha + \beta - \alpha \cdot \beta$	$\begin{cases} 1, & \alpha = 0 \\ 0, & \alpha > 0 \end{cases}$	$\begin{cases} 1 & \alpha \leqslant \beta \\ \beta / \alpha, & \alpha > \beta \end{cases}$

- Zadeh family can be represented using Łukasiewicz:
  - negation $\neg$: $\ominus \alpha$
  - t-norm $\wedge$: $\alpha \otimes (\alpha \Rightarrow \beta)$
  - t-conorm $\vee$: $\neg((\neg\alpha) \wedge (\neg\beta))$
  - Implication: $(\neg\alpha) \vee \beta$

Why to use Łukasiewicz logic?

- Łukasiewicz logic is **more general** than Zadeh family.
- In some applications, Zadeh family is not enough. Example: **Fuzzy matchmaking**.
  - A concept *Buy* collects all the **buyer's preferences** together in such a way that the higher is the maximal degree of satisfiability, the more the buyer is satisfied, e.g.,

$$Buy \equiv Sedan \sqcap (\exists hasPrice.about30000) \sqcap (\exists hasColor.Black)$$

  - A concept *Sell* collects all the **seller's preferences** together in such a way that the higher is the maximal degree of satisfiability, the more the seller is satisfied.
  - The **best agreement** between them is determined by the maximal degree of satisfiability of the conjunction $Buy \sqcap Sell$.
  - Using Łukasiewicz t-norm the solution is **Pareto optimal**.
  - But this does not hold for the minimum t-norm (Zadeh family)!!

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Vocabulary

- Vocabulary of the language:
  - **Individuals**: *fernando*.
  - **Concepts**, fuzzy sets of individuals: *Tall*.
  - **Roles**, fuzzy binary relations over individuals: *isFriendOf*.
- **Complex definitions** of concepts and roles can be built.
- A **fuzzy interpretation** $\mathcal{I}$ is a pair $(\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$.
  - $\Delta^{\mathcal{I}}$ is a non empty set, the interpretation domain
  - $\cdot^{\mathcal{I}}$ is an interpretation function mapping:
    - Each concept onto a function $C^{\mathcal{I}} : \Delta^{\mathcal{I}} \to [0, 1]$
    - Each role onto a function $R^{\mathcal{I}} : \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \to [0, 1]$
- A **fuzzy knowledge base** consists of fuzzy axioms organized in:
  - A fuzzy **ABox** $\mathcal{A}$: extensional knowledge about individuals.
  - A fuzzy **TBox** $\mathcal{T}$: intensional knowledge about concepts.
  - A fuzzy **RBox** $\mathcal{R}$: intensional knowledge about roles.
- Most reasoning tasks are reducible to **KB consistency**.

Fuzzy concepts and axioms

Fuzzy concepts	Syntax	Semantics
top	$\top$	1
bottom	$\bot$	0
atomic concept	*Blond*	$A^{\mathcal{I}}(a)$
concept conjunction	*Human* $\sqcap$ *Blond*	$C^{\mathcal{I}}(a) \otimes D^{\mathcal{I}}(a)$
concept disjunction	*Clever* $\sqcup$ *Blond*	$C^{\mathcal{I}}(a) \oplus D^{\mathcal{I}}(a)$
concept negation	$\neg$*Blond*	$\ominus C^{\mathcal{I}}(a)$
universal quantification	$\forall hasChild.Human$	$\inf_{b \in \Delta^{\mathcal{I}}} \{R^{\mathcal{I}}(a,b) \Rightarrow C^{\mathcal{I}}(b)\}$
existential quantification	$\exists hasChild.Woman$	$\sup_{b \in \Delta^{\mathcal{I}}} \{R^{\mathcal{I}}(a,b) \otimes C^{\mathcal{I}}(b)\}$
at-least cardinality	$\geqslant 3 \ hasChild.Blond$	...
at-most cardinality	$\leqslant 2 \ hasChild.Blond$	...

Fuzzy axioms	Syntax	Semantics
concept assertion	$\langle fernando : Human \bowtie \gamma \rangle$	$C^{\mathcal{I}}(a^{\mathcal{I}}) \bowtie \gamma$
role assertion	$\langle (fernando, umberto) : isFriendOf \bowtie \gamma \rangle$	$R^{\mathcal{I}}(a^{\mathcal{I}}, b^{\mathcal{I}}) \bowtie \gamma$
general concept inclusion	$\langle Inn \sqsubseteq Hotel \bowtie \gamma \rangle$	$\inf_{a \in \Delta^{\mathcal{I}}} \{C^{\mathcal{I}}(a) \Rightarrow D^{\mathcal{I}}(a)\} \bowtie \gamma$

Cardinality restrictions

- **Most common semantics** in fuzzy DLs for cardinality restrictions:

$$(\geqslant n\ R.C)^{\mathcal{I}}(a) = \sup_{b_1,\ldots,b_n \in \Delta^{\mathcal{I}}} [(\otimes_{i=1}^{n} \{R^{\mathcal{I}}(a,b_i) \otimes C^{\mathcal{I}}(b_i)\}) \bigotimes (\otimes_{j<k} \{b_j \neq b_k\})]$$
$$(\leqslant n\ R.C)^{\mathcal{I}}(a) = \inf_{b_1,\ldots,b_{n+1} \in \Delta^{\mathcal{I}}} [(\otimes_{i=1}^{n+1} \{R^{\mathcal{I}}(a,b_i) \otimes C^{\mathcal{I}}(b_i)\}) \Rightarrow (\oplus_{j<k} \{b_j = b_k\})]$$

- It derives from the classical case, by deriving the concept $(\geqslant n\ R.C)$ as the first-order formula

$$\exists x_1, \ldots x_n . (\bigwedge_i R(x, x_i) \wedge C(x_i)) \wedge (\bigwedge_{i<j} x_i \neq x_j)$$

  and assuming that $(\leqslant n\ R.C)$ is the same as $\neg(\geqslant n+1\ R.C)$.

- However, the semantics may be **counter-intuitive**, as it will be shown in the following example.

Cardinality restrictions

Example

Assume the following model:

- $((\mathit{fernando}, \mathit{apple}) : \mathit{likes})^{\mathcal{I}} = 0.5$
- $((\mathit{fernando}, \mathit{banana}) : \mathit{likes})^{\mathcal{I}} = 0.5$
- $((\mathit{fernando}, \mathit{orange}) : \mathit{likes})^{\mathcal{I}} = 0.5$
- $((\mathit{fernando}, \mathit{peach}) : \mathit{likes})^{\mathcal{I}} = 0.5$
- $(\mathit{apple} : \mathit{Fruit})^{\mathcal{I}} = 1$
- $(\mathit{banana} : \mathit{Fruit})^{\mathcal{I}} = 1$
- $(\mathit{orange} : \mathit{Fruit})^{\mathcal{I}} = 1$
- $(\mathit{peach} : \mathit{Fruit})^{\mathcal{I}} = 1$
- $\mathit{apple}^{\mathcal{I}}, \mathit{banana}^{\mathcal{I}}, \mathit{orange}^{\mathcal{I}}, \mathit{peach}^{\mathcal{I}}$ are mutually different.

Then $(\leqslant 1\ \mathit{likes}.\mathit{Fruit})^{\mathcal{I}}(\mathit{fernando}) = 1$. *fernando* has 4 fillers and could have many more such that $((\mathit{fernando}, x_i) : \mathit{likes})^{\mathcal{I}} + (x_i : \mathit{Fruit})^{\mathcal{I}} \leqslant 1$.

Cardinality restrictions

- We argue that the following **properties** should be satisfied:

Property

1. *If* $(\leqslant n\ R.C)^{\mathcal{I}}(a) = 1$ *then* $\sharp\{b \mid (R(a,b)^{\mathcal{I}} \otimes C(b))^{\mathcal{I}} > 0\} \leqslant n$.
2. $\exists R.C \equiv \geqslant 1 R.C$.
3. $\leqslant n\ R.C \equiv \neg(\geqslant n+1\ R.C)$.

- Property 1 requires that $\otimes_{i=1}^{n+1}\{R^{\mathcal{I}}(a,b_i) \otimes C^{\mathcal{I}}(b_i)\} > 0$ if (and only if) $R^{\mathcal{I}}(a,b_i) \otimes C^{\mathcal{I}}(b_i) > 0$ for every $i \in \{1,\ldots,n+1\}$. But Łukasiewicz t-norm does not verify it.
- As a solution, we propose a new semantics:

$$(\geqslant n\ R.C)^{\mathcal{I}}(a) = \sup_{b_1,\ldots,b_n \in \Delta^{\mathcal{I}}}[\min_{i=1}^{n}\{R^{\mathcal{I}}(a,b_i) \otimes C^{\mathcal{I}}(b_i)\} \bigotimes (\otimes_{j<k}\{b_j \neq b_k\})]$$
$$(\leqslant n\ R.C)^{\mathcal{I}}(a) = \inf_{b_1,\ldots,b_{n+1} \in \Delta^{\mathcal{I}}}[\min_{i=1}^{n+1}\{R^{\mathcal{I}}(a,b_i) \otimes C^{\mathcal{I}}(b_i)\} \Rightarrow (\oplus_{j<k}\{b_j = b_k\})]$$

Related work

- The most common semantics was introduced by **G. Stoilos et al.** as a modification of a previous definition by **U. Straccia**.
- **D. Sánchez et al.** proposed the use of fuzzy quantifiers in $\mathcal{ALCQ}$, making possible to express e.g. that a customer *mostly* buys cheap items, but:
  - Reasoning becomes particularly harder.
  - Their approach strongly relies on finite models, which is a problem for more expressive logics.
- **S. Calegari et al.** introduced another semantics for unqualified cardinality restrictions (a special case where $C = \top$).
  - $\geqslant n\ R$ and $\leqslant n\ R$ are crisp concepts. $\geqslant n\ R$ is interpreted as "the individual has at least n R-successors with a degree greater than 0" ($\leqslant n\ R$ is interpreted dually).
  - Property 2 does not hold.

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Basic idea

- **Mixture of DL tableaux rules and bMILP**.
- Firstly, some **tableaux rules** are applied.
  - We assume that concepts are in Negation Normal Form (NNF).
  - These rules generate simpler concept expressions and some (linear) constraints.
  - Rules which generate new individuals –$(\exists), (\geqslant n)$– are applied as last.
  - We use a blocking condition to detect cycles in fuzzy GCIs.
- Finally, one or several **bMILP problems** on the set of generated constraints are solved.
  - Rules (disjunction and GCI rules) do not have to guess as it is the case of crisp DLs and some fuzzy DLs.
- We consider the fuzzy operators from Łukasiewicz logic but similar ideas also apply to Zadeh and classical logics!

Example: $\sqcap$-rule

- Recall the semantics of the conjunction:
  $(C_1 \sqcap C_2)^{\mathcal{I}}(x) = \max\{C_1{}^{\mathcal{I}}(x) + C_2{}^{\mathcal{I}}(x) - 1, 0\}$

- **Informally**: If $a \in \langle Tall \sqcap Fat, 0.5\rangle$ then:
  - create two variables x_1, x_2,
  - add the assertion $a \in \langle Tall, x_1\rangle$,
  - add the assertion $a \in \langle Fat, x_2\rangle$,
  - add the restriction $\max\{x_1 + x_2 - 1, 0\} = 0.5$,
  - add the restrictions $x_1 \in [0, 1], x_2 \in [0, 1]$.

- **Formally** the rule is: If $\langle C \sqcap D, l\rangle \in \mathcal{L}(v)$ then append $\langle C, x_1\rangle$ and $\langle D, x_2\rangle$ to $\mathcal{L}(v)$, and $\mathcal{C_F} = \mathcal{C_F} \cup\ y \leqslant 1 - l, x_i \leqslant 1 - y,$ $x_1 + x_2 = l + 1 - y, x_i \in [0, 1], y \in \{0, 1\}$, where x_i, y are new variables.
  - if $y = 0$ then $x_1 + x_2 - 1 = l$,
  - if $y = 1$ then $l = x_1 = x_2 = 0$.

New rules

($\geqslant$). If $\langle \geqslant n\ R.C, l \rangle \in \mathcal{L}(v)$, v is not blocked, the rule has not yet been applied to v, then: create n new nodes $w_1 \ldots w_n$ with $\langle R, r_i \rangle$ to $\mathcal{L}(\langle v, w_i \rangle)$, $\langle C, c_i \rangle$ to $\mathcal{L}(w_i)$, $w_i \neq w_j$ and $\mathcal{C_F} = \mathcal{C_F} \cup \{y_i \leqslant 1 - l, r_i \leqslant 1 - y_i, c_i \leqslant 1 - y_i, r_i + c_i = l + 1 - y_i, c_i \in [0, 1], r_i \in [0, 1], y_i \in \{0, 1\}\}$, where c_i, r_i, y_i are new variables.

(ch). If $\langle \bowtie n\ R.C, l \rangle \in \mathcal{L}(v)$, $\bowtie \in \{\leqslant, \geqslant\}$ and there is an R-successor w of v such that the rule has not yet been applied to v and w, then append $\mathcal{L}(w) = \mathcal{L}(w) \cup \{ \langle C, x \rangle, \langle \neg C, 1 - x \rangle \}$ and $\mathcal{C_F} = \mathcal{C_F} \cup \{x \in [0, 1]\}$, where x is a new variable.

($\leqslant$). If $\langle \leqslant n\, R.C, l \rangle \in \mathcal{L}(v)$ and there are $n+1$ R-successors $w_1, \ldots, w_{n+1}$ of v with $\langle C, l_i \rangle \in \mathcal{L}(w_i)$, $\langle R, r_i \rangle \in \mathcal{L}(\langle v, w_i \rangle)$, then non-deterministically apply one of the following subrules:

1. $\mathcal{C}_{\mathcal{F}} = \mathcal{C}_{\mathcal{F}} \cup \{l = 0\}$.
2. Append $\langle \neg C_i, 1 - x_i \rangle$ to $\mathcal{L}(w_i)$ and $\mathcal{C}_{\mathcal{F}} = \mathcal{C}_{\mathcal{F}} \cup \{$ $x_{(v,\, w_i):R} + x_i + y_i \leqslant 2 - l$, $y_1 + \cdots + y_{n+1} = n$, $x_i \in [0, 1]$, $y_i \in \{0, 1\}\}$, where x_i, y_i are new variables.
3. For every pair of individuals w_i and w_j, $1 \leqslant i < j \leqslant n$, such that w_j is not an ancestor of w_i and not $w_i \neq w_j$, apply *Merge*(v, w_i, w_j).
4. If for all pairs $w_i, w_j, 1 \leqslant i < j \leqslant n, w_i \neq w_j$, then we have an inconsistency, so add $\mathcal{C}_{\mathcal{F}} = \mathcal{C}_{\mathcal{F}} \cup \{0 = 1\}$.

Intuition of the new rules

- $(\geqslant)$ creates new n R-successors in case they do not exist, in such a case that the semantics of the constructor is satisfied.
- (ch) states that w belongs to C and $\neg C$ to some degree, and we know that $C^{\mathcal{I}}(x) + (\neg C)^{\mathcal{I}}(x) = C^{\mathcal{I}}(x) + 1 - C^{\mathcal{I}}(x) = 1$.
  - As opposed to the crisp case, the rule is deterministic.
- $(\leqslant)$ is more tricky (see next slide).
- Without the $\leqslant n\ R.C$ construct, the generated tableaux is deterministic and, thus, just **one bMILP problem** has to be solved.
- This is no longer true once we introduce cardinality restrictions: due to the $(\leqslant)$ rule, **several bMILP problems** may need to be solved in order to determine whether the KB is satisfiable or not.
  - In order to find the minimum solution, in fact, it is necessary to solve **all** of them.

Intuition of the new rules

- $(\leqslant)$ rule guarantees that there do not exist $n+1$ R-successors, which leads to **several cases**:
  1. If $l = 0$ then we have $\langle \leqslant n\ R.C, 0 \rangle$, which is a tautology.
  2. The minimum over all $R^{\mathcal{I}}(v, w_i) \otimes C^{\mathcal{I}}(w_i) \geqslant l$ is less or equal than $1 - l$. That is, there is an R-successor w_i satisfying this. The constraints on the control variables y_i require that exactly one of them takes the value 0. If a control variable takes the value 1, it does not impose any restriction. Otherwise, if $y_i = 0$ then $x_{(v, w_i):R} \otimes x_i \leqslant 1 - l$ and, together with the assertion $\langle \neg C_i, 1 - x_i \rangle$ to $\mathcal{L}(v)$, this guarantees that $R^{\mathcal{I}}(v, w_i) \otimes C^{\mathcal{I}}(w_i) \leqslant 1 - l$.
  3. Two successors may be interpreted as the same individual, so we merge them into one equivalent individual.
  4. No individual can be merged (for all possible pairs of individuals, they are required to be different) but $l \neq 0$, so consequently the KB is inconsistent.

Properties of the algorithm

Proposition

1. *A KB $\mathcal{K}$ is satisfiable iff there exists a fuzzy tableau for $\mathcal{K}$.*
2. **Termination**. *For each KB $\mathcal{K}$, the tableau algorithm terminates.*
3. **Soundness**. *If the expansion rules can be applied to a KB $\mathcal{K}$ such that they yield a complete completion-forest $\mathcal{F}$ such that $\mathcal{C}_{\mathcal{F}}$ has a solution, then $\mathcal{K}$ has a fuzzy tableau for $\mathcal{K}$.*
4. **Completeness**. *Consider a KB $\mathcal{K}$. If $\mathcal{K}$ has a fuzzy tableau, then the expansion rules can be applied in such a way that the tableaux algorithm yields a complete completion-forest for $\mathcal{K}$ such that $\mathcal{C}_{\mathcal{F}}$ has a solution.*

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Conclusions and Future Work

- **Conclusions**:
  - We have proposed a **novel semantics** for qualified cardinality restrictions.
  - We have presented a reasoning algorithm for fuzzy $\mathcal{ALCQ}$ under Łukasiewicz semantics.
  - Adding cardinality restrictions makes reasoning **more difficult** because the algorithm needs to solve several optimization problems.
  - Zadeh family can be represented using Łukasiewicz logic, so our use of cardinality restrictions is more general than previous work.
- **Future work**:
  - Extend the **expressivity** of the logic towards fuzzy $\mathcal{SHIF}(\mathbf{D})$ (fuzzy OWL Lite) and fuzzy $\mathcal{SHOIN}(\mathbf{D})$ (fuzzy OWL DL).
  - Implementation of the algorithm in the **fuzzyDL** reasoner.

Questions?

Thank you very much for your attention