Multi Criteria Decision Making in Fuzzy Description Logics: A First Step

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Abstract. Fuzzy Description Logics are logics which allow to deal with structured knowledge affected by vagueness. Although a relatively important amount of work has been carried out in the last years, fuzzy DLs are open to be extended with several features worked out in other fields. In this work, we start addressing the problem of incorporating Multi-Criteria Decision Making (MCDM) into fuzzy Description Logics and, thus, start an investigation about offering the possibility of a fuzzy ontology assisted approach to decision making.

1 Introduction

Description Logics (DLs) [1] play a key role in the design of *Ontologies*. An ontology consists of a hierarchical description of important concepts in a particular domain, along with the description of the properties (of the instances) of each concept. DLs play a particular role in this context as they are essentially the theoretical counterpart of the *Web Ontology Language OWL DL*, a state of the art language to specify ontologies.

It is well-known that "classical" ontology languages are not appropriate to deal with *fuzzy knowledge*, which is inherent to several real world domains [10,12]. Fuzzy ontologies emerge as useful in several applications, such as (multimedia) information retrieval, image interpretation, ontology mapping, matchmaking and the Semantic Web [8]. So far, several fuzzy extensions of DLs can be found in the literature (see the survey in [8]) and some fuzzy DL reasoners have been implemented, such as FUZZYDL [3], DELOREAN [2] or FIRE [9].

In this work, we start investigating about using fuzzy DLs as a fuzzy ontology support for *Multi-Criteria Decision Making* (MCDM) [13], which is among one of the most well known branches of decision making. Roughly, MCDM is the study of identifying and choosing alternatives based on the values and preferences of the decision maker. Making a decision implies that there are alternative choices to be considered and to choose the one that best fits with our goals, objectives, desires, values, and so on. Our work should be understood as an attempt in using fuzzy DLs and, thus, fuzzy ontologies, for knowledge assisted decision making. While there is a large literature on fuzzy MCDM [6] and fuzzy DLs [8], to the best of our knowledge, this is the first time such a combination is addressed.

We proceed as follows. Section 2 (resp. Section 3) will provide the basic concepts related to mathematical fuzzy logic (resp. MCDM) we will rely on, Section 4

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specifies a minimal fuzzy DL to deal with MCDM and illustrates some examples. Section 5 concludes and describes some future work.

2 Fuzzy Sets and Mathematical Fuzzy Logic Basics

In *Mathematical Fuzzy Logic* [5], the convention prescribing that a statement is either true or false is changed and is a matter of degree measured on an ordered scale $\mathcal{S}$ that is no longer $\{0,1\}$, but usually the unit interval $[0,1]$. This degree of fit is called *degree of truth* of the statement ϕ in the interpretation $\mathcal{I}$. In this section, for illustrative purposes, *fuzzy statements* have the form $\phi \geqslant l$ or $\phi \leqslant u$, where $l, u \in [0,1]$ (see, e.g. [5]) and ϕ is a statement. Fuzzy statements encode that the degree of truth of ϕ is *at least equal to* l resp. *at most equal to* u. A *fuzzy interpretation* $\mathcal{I}$ maps each basic statement p_i into $[0,1]$ and is then extended inductively to all statements: $\mathcal{I}(\phi \land \psi) = \mathcal{I}(\phi) \otimes \mathcal{I}(\psi)$, $\mathcal{I}(\phi \lor \psi) = \mathcal{I}(\phi) \oplus \mathcal{I}(\psi)$, $\mathcal{I}(\phi \to \psi) = \mathcal{I}(\phi) \Rightarrow \mathcal{I}(\psi)$, $\mathcal{I}(\neg\phi) = \ominus\mathcal{I}(\phi)$, $\mathcal{I}(\exists x.\phi(x)) = \sup_{a \in \Delta^{\mathcal{I}}} \mathcal{I}(\phi(a))$, $\mathcal{I}(\forall x.\phi(x)) = \inf_{a \in \Delta^{\mathcal{I}}} \mathcal{I}(\phi(a))$, where $\Delta^{\mathcal{I}}$ is the domain of $\mathcal{I}$, and $\otimes$, $\oplus$, $\Rightarrow$, and $\ominus$ are *t-norms*, *t-conorms*, *implication functions*, and *negation functions*, respectively, which extend the classical Boolean conjunction, disjunction, implication, and negation, respectively, to the fuzzy case. One usually distinguishes three different logics (see below), namely Łukasiewicz, Gödel, and Product logic [5]. Zadeh "logic", namely $a \otimes b = \min(a,b)$, $a \oplus b = \max(a,b)$, $\ominus a = 1 - a$ and $a \Rightarrow b = \max(1-a, b)$ is entailed by Łukasiewicz logic.

	Łukasiewicz Logic	Gödel Logic	Product Logic
$a \otimes b$	$\max(a+b-1, 0)$	$\min(a,b)$	$a \cdot b$
$a \oplus b$	$\min(a+b, 1)$	$\max(a,b)$	$a + b - a \cdot b$

	Łukasiewicz Logic	Gödel Logic	Product Logic
$a \Rightarrow b$	$\min(1-a+b, 1)$	$\begin{cases} 1 & \text{if } a \leq b \\ b & \text{otherwise} \end{cases}$	$\min(1, b/a)$
$\ominus a$	$1 - a$	$\begin{cases} 1 & \text{if } a = 0 \\ 0 & \text{otherwise} \end{cases}$	$\begin{cases} 1 & \text{if } a = 0 \\ 0 & \text{otherwise} \end{cases}$

In *fuzzy set theory* [7], a *fuzzy set* R over a countable crisp set X is a function $R\colon X \to [0,1]$. A (binary) *fuzzy relation* R over two countable crisp sets X and Y is a function $R\colon X \times Y \to [0,1]$. We say that R is *functional* iff R is a partial function $R\colon X \times Y \to \{0,1\}$ such that for each $x \in X$ there is unique $y \in X$ where $R(x,y)$ is defined. The *degree of subsumption* between two fuzzy sets A and B is defined as $\inf_{x \in X} A(x) \Rightarrow B(x)$ and may be seen as the degree of the FOL formula $\forall x.A(x) \to B(x)$, while the *degree of overlap* between two fuzzy sets A and B is defined as $\sup_{x \in X} A(x) \land B(x)$ and may be seen as the degree of the FOL formula $\exists x.A(x) \land B(x)$.

The notions of satisfiability and logical consequence are defined in the standard way. A fuzzy interpretation $\mathcal{I}$ *satisfies* a fuzzy statement $\phi \geqslant l$ (resp., $\phi \leqslant u$) or $\mathcal{I}$ is a *model* of $\phi \geqslant l$ (resp., $\phi \leqslant u$), denoted $\mathcal{I} \models \phi \geqslant l$ (resp., $\mathcal{I} \models \phi \leqslant u$), iff $\mathcal{I}(\phi) \geqslant l$ (resp., $\mathcal{I}(\phi) \leqslant u$). Furthermore, $\phi \geqslant l$ is a *tight logical consequence* of a set of fuzzy statements $\mathcal{K}$ iff l is the infimum of $\mathcal{I}(\phi)$ subject to all models $\mathcal{I}$ of $\mathcal{K}$. The latter value is equivalent to $l = \sup\{r \mid \mathcal{K} \models \phi \geqslant r\}$, it is called *Best Entailment Degree* (BED), and is denoted $bed(\mathcal{K}, \phi)$, while the *Best Satisfiability Degree* (BSD), denoted as $bsd(\mathcal{K}, \phi)$, is defined as $\sup_{\mathcal{I} \models \mathcal{K}} \mathcal{I}(\phi)$.

3 MCDM Basics

The area of MCDM is quite vast and we cannot address all the addressed issues here. We will focus on the basic notions that are of importance in MCDM and a simple MCDM method to be used here.

Usually, *alternatives* represent different choices of action available to the decision maker and is assumed to be finite in our case. The *decision criteria* represent the different dimensions from which the alternatives can be viewed (a decision criteria is also referred to as *goals* or *attributes*). Most of the MCDM methods require the criteria be assigned *decision weights* of importance. Usually, these weights are normalized to add up to one.

A *MCDM problem* of m criteria and n alternatives is informally as follows: let $\mathbf{A} = \{A_1, \ldots, A_n\}$ be a set of n decision alternatives and let $\mathbf{C} = \{C_1, \ldots, C_m\}$ be a set of m criteria according to which the desirability of an action is judged. Determine the optimal alternative A^* with the highest degree of desirability.

A standard feature of MCDM methods is that a MCDM problem can be expressed by means of a *decision matrix*, as shown below.

		Criteria				
		w_1	w_2	·	·	w_m
Alternatives		C_1	C_2	·	·	C_m
x_1	A_1	a_{11}	a_{12}	·	·	a_{1m}
x_2	A_2	a_{21}	a_{22}	·	·	a_{2m}
·	·	·	·	·	·	·
·	·	·	·	·	·	·
x_n	A_n	a_{n1}	a_{n2}	·	·	a_{nm}

(1)

In the matrix each column belongs to a criterion C_j and each row describes the performance of an alternative A_i. The score a_{ij} describes the performance of alternative A_i against criterion C_j. The weights $w_1, \ldots, w_m$ are assigned to the criteria. Weight w_j reflects the relative importance of criteria C_j to the decision, and is assumed to be positive and normalized, i.e. $1 = \sum_{j=1}^{m} w_j$. The weights of the criteria are usually determined on subjective basis and may also be seen as a kind of profit of the criteria. They represent the opinion of a single decision maker or synthesize the opinions of a group of experts. Not surprisingly, there is a large literature on methods to assign weights (see, e.g. [13]). For illustrative purpose, we illustrate here a method based on pairwise comparison of the criteria to determine the weights. It consists in comparing elements (criteria) X_i with X_j ($1 \leq i, j \leq k$) and judge how much they contribute to the overall objective. The judgment consists in assigning a number $w_{ij} \in [1, 9]$, called *Intensity of Importance*, selected according the following table

Intensity	Definition	Explanation
1	Equal Importance	Two elements contribute equally to the objective
3	Moderate Importance	Experience and judgement slightly favor one element over the other
5	Strong Importance	Experience and judgement strongly favor one element over the other
7	Very Strong Importance	One element is favored very strongly over another, its dominance is demonstrated in practice
9	Extreme Importance	The evidence favoring one element over another is of the highest possible order of information

Intermediate values can be used. The weight w_i of element X_i may be obtained as

$$\bar{w_i} = (\prod_{j=1}^{k} w_{ij})^{1/k} \ , \ \bar{w} = \sum_{i=1}^{k} \bar{w_i} \ , \ \text{ and then } w_i = \bar{w_i}/\bar{w} \ .$$

On more on alternatives to determine the data of a decision matrix and their impact, see e.g., [13].

The values $x_1, \ldots, x_n$ associated with the alternatives in the decision matrix will be used to denote the final ranking values of the alternatives. Usually, higher ranking value means a better performance of the alternative, so the alternative with the highest ranking value is the best of the alternatives. MCDM techniques can partially or completely rank the alternatives: a single most preferred alternative can be identified or a short list of a limited number of alternatives can be selected for subsequent detailed appraisal. Again, there are many alternative methods to compute the final ranking values from the decision matrix. For illustrative purposes, we present the so-called *Weighted Sum Method* (WSM), which is among the simplest methods in MCDM, but has the advantage to be easy embedded within fuzzy DLs. Formally, let

$$x_i = \sum_{j=1}^{m} a_{ij} w_j \ \text{ for } i = 1, 2, \ldots, m \ . \tag{2}$$

where x_i is the the *final ranking value* of alternative A_i. The *ranking of the alternatives* is obtained by ordering the alternatives in descending order with respect to the final ranking value and the *optimal alternative* A^* is the one that maximizes the final ranking value, i.e.

$$A^* = \arg \max_{A_i} x_i \ .$$

We conclude this section by pointing out that in *fuzzy* MCDM, a principal difference to classical MCDM is due to the fact that weights w_i and performance factors a_{ij} are so-called *fuzzy numbers* [7]. A fuzzy number $\tilde{n}$ is a fuzzy set over relas with triangular membership function $tri(a, b, c)$ and is intended being an approximation of the number b. Any real value n is seen as the fuzzy number $tri(n, n, n)$. The arithmetic operators $+, -, \cdot$ and $\div$ are extended to fuzzy numbers by applying them to the arguments, i.e. for fuzzy numbers $\tilde{n}_1 = tri(a_1, b_1, c_1)$ and $\tilde{n_2} = tri(a_2, b_2, c_2)$, for operator $* \in \{+, \cdot\}$, $\tilde{n}_1 * \tilde{n}_2 = tri(a_1 * a_2, b_1 * b_2, c_1 * c_2)$, while for $* \in \{-, \div\}$, $\tilde{n}_1 * \tilde{n}_2 = tri(a_1 * c_2, b_1 * b_2, c_1 * a_2)$. The final rank value is computed as in Eq. 2,

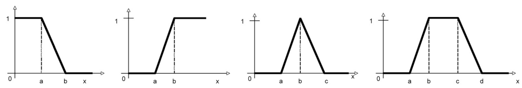
$$\tilde{x}_i = \sum_{j=1}^{m} \tilde{a}_{ij} \cdot \tilde{w}_j \ \text{ for } i = 1, 2, \ldots, m \ . \tag{3}$$

As now $\tilde{x}_i$ is a fuzzy number, one may apply some defuzzyfication method to it and compere fuzzy numbers based on these values, or one may use some fuzzy number comparison operator to determine the optimal solution (see, e.g. [13]).

4 Towards MCDM within Fuzzy Description Logics

The aim of this section is to show that current fuzzy DLs can be used to deal with (fuzzy) knowledge assisted MCDM (though, the MCDM method needs to be simple so far). For illustrative purposes and for reasons of space, we will just consider a minimal fuzzy DL to deal with the WSM in MCDM.

Syntax. We will present $\mathcal{ALCF}(D)$, which is the basic DL $\mathcal{ALC}$ extended with functional roles (letter $\mathcal{F}$) and concrete domains [11] (letter D).



In general, a *fuzzy concrete domain* (or simply *fuzzy domain*) [11] is a pair $\langle \Delta_D, \Phi_D \rangle$, where Δ_D is an interpretation domain and Φ_D is the set of *fuzzy domain predicates* d with a predefined arity n and an interpretation $d^D : \Delta_D^n \to [0, 1]$, which is a n-ary fuzzy relation over Δ_D. In our specific spatial fuzzy DL, we assume that predicates are unary and Δ_D are non-negative real numbers.

Now, consider pairwise disjoint alphabets of *concepts names* (denoted A), *abstract roles names* (denoted R) and *concrete roles names* (denoted T). Within the alphabet of abstract and concrete roles, we have distinguished subsets of *abstract functional roles names* (denoted f) and *concrete functional roles names* (denoted t), respectively. We call functional roles also *features*. From a First-Order Logic point of view, concepts may be seen as a formulae with one free variable (and, thus, may be seen as class descriptors), while roles as binary predicates (and, thus, may be used to describe properties of a class). *Concepts* (denoted C or D) of the language can be built inductively from atomic concepts (A), top concept $\top$, bottom concept $\bot$, abstract roles (R), concrete roles (T) as follows. The syntax of fuzzy concepts (denoted C, D) is as follows:

$$C, D := \top \mid \bot \mid A \mid C \sqcap D \mid C \sqcup D \mid \neg C \mid \forall R.C \mid \exists R.C$$

Now, the fuzzy DL is extended as follows [3]:

$$\begin{array}{ll} C, D & := \forall T.d \mid \exists T.d \mid w_1 C_1 + \cdots + w_k C_k \\ d & := ls(a, b) \mid rs(a, b) \mid tri(a, b, c) \mid trap(a, b, c, d) \end{array}$$

where *val* is an integer, a real or a string depending on the range of the concrete feature t, $w_i \in [0, 1]_D, \sum_{i=1}^{k} w_i \leq 1$ and C_i are concepts[1]. E.g., the expression $Human \sqcap (\leq hasAge\ 18)$ will denote the set of humans, which have an age less or equal than 18, while $Human \sqcap \exists hasAge.ls(10, 30)$ will denote the set of young humans (their age is $ls(10, 30)$).

A *Fuzzy Knowledge Base* (or *fuzzy Ontology*) consists of a finite set of *fuzzy General Concept Inclusions (fuzzy GCIs)*, which are expressions of the form $\langle C \sqsubseteq D, n \rangle$ (with informal meaning, the degree of subsumption between concept C and D is not less than n). In FOL, $\langle C \sqsubseteq D, n \rangle$ may be seen as a fuzzy statement of the form $(\forall x.C(x) \to D(x)) \geqslant n$ and amounts of asserting that the

[1] In [3] we assume $\sum_{i=1}^{k} w_i = 1$ instead, however, this modification is harmless.

degree of subsumption among C and D is at least n. We will use $C = D$ as a shorthand for $\langle C \sqsubseteq D, 1\rangle$ and $\langle D \sqsubseteq C, 1\rangle$.

Semantics. From a semantics point of view, a *fuzzy interpretation* $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ relative to the fuzzy concrete domain $\langle \Delta_D, \Phi_D\rangle$, consists of a nonempty set $\Delta^{\mathcal{I}}$ (the *domain*), disjoint from Δ_D, and of a *fuzzy interpretation function* $\cdot^{\mathcal{I}}$ that coincides with $\cdot_D$ on every fuzzy concrete predicate, and it assigns: (i) to each abstract concept C a function $C^{\mathcal{I}}\colon \Delta^{\mathcal{I}} \to [0, 1]$; ($ii$) to each abstract role R a function $R^{\mathcal{I}}\colon \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \to [0, 1]$; ($iii$) to each abstract feature r a partial function $r^{\mathcal{I}}\colon \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \to \{0, 1\}$ such that for all $u \in \Delta^{\mathcal{I}}$ there is an unique $w \in \Delta^{\mathcal{I}}$ on which $r^{\mathcal{I}}(u, w)$ is defined; (iv) to each concrete role T a function $T^{\mathcal{I}}\colon \Delta^{\mathcal{I}} \times \Delta_D \to [0, 1]$; ($v$) to each concrete feature t a partial function $t^{\mathcal{I}}\colon \Delta^{\mathcal{I}} \times \Delta_D \to \{0, 1\}$ such that for all $u \in \Delta^{\mathcal{I}}$ there is an unique $r \in \Delta_D$ on which $t^{\mathcal{I}}(u, r)$ is defined.

Given arbitrary t-norm $\otimes$, t-conorm $\oplus$, negation function $\ominus$ and implication function $\Rightarrow$, the fuzzy interpretation function is extended to *complex concepts* and *fuzzy axioms* as below:

$$
\begin{aligned}
(\top)^{\mathcal{I}}(x) &= 1\\
(\bot)^{\mathcal{I}}(x) &= 0\\
(A)^{\mathcal{I}}(x) &= A^{\mathcal{I}}(x)\\
(C \sqcap D)^{\mathcal{I}}(x) &= C^{\mathcal{I}}(x) \otimes D^{\mathcal{I}}(x)\\
(C \sqcup D)^{\mathcal{I}}(x) &= C^{\mathcal{I}}(x) \oplus D^{\mathcal{I}}(x)\\
(\neg C)^{\mathcal{I}}(x) &= \ominus C^{\mathcal{I}}(x)\\
(\forall R.C)^{\mathcal{I}}(x) &= \inf_{y \in \Delta^{\mathcal{I}}}\{R^{\mathcal{I}}(x, y) \Rightarrow C^{\mathcal{I}}(y)\}
\end{aligned}
$$

$$
\begin{aligned}
(\exists R.C)^{\mathcal{I}}(x) &= \sup_{y \in \Delta^{\mathcal{I}}}\{R^{\mathcal{I}}(x, y) \otimes C^{\mathcal{I}}(y)\}\\
(\forall T.\mathbf{d})^{\mathcal{I}}(x) &= \inf_{r \in \Delta_D}\{T^{\mathcal{I}}(x, r) \Rightarrow \mathbf{d}^{\mathcal{I}}(r)\}\\
(\exists T.\mathbf{d})^{\mathcal{I}}(x) &= \sup_{r \in \Delta_D}\{T^{\mathcal{I}}(x, r) \otimes \mathbf{d}^{\mathcal{I}}(r, r')\}\\
(w_1 C_1 + \cdots + w_k C_k)^{\mathcal{I}}(x) &= w_1 {C_1}^{\mathcal{I}}(x) + \cdots + w_k {C_k}^{\mathcal{I}}(x)\\
(C \sqsubseteq D)^{\mathcal{I}} &= \inf_{x \in \Delta^{\mathcal{I}}}\{C^{\mathcal{I}}(x) \Rightarrow D^{\mathcal{I}}(x)\}
\end{aligned}
$$

A fuzzy interpretation $\mathcal{I}$ *satisfies* (is a *model* of) a fuzzy statement $\langle \alpha, n\rangle$ iff $\alpha^{\mathcal{I}} \geq n$. The notions of logical consequence, best entailment degree and best satisfiability degree of α are as for Section 2. We additionally define the *Best Satisfiability Degree* (BSD) [3] of a concept C w.r.t. a fuzzy KB $\mathcal{K}$ as

$$bsd(\mathcal{K}, C) = \sup_{\mathcal{I} \models \mathcal{K}} \sup_{x \in \Delta^{\mathcal{I}}} C^{\mathcal{I}}(x) \ .$$

MCDM and Fuzzy DLs. We next provide some examples, illustrating how to encode some simple (fuzzy) MCDM problems in fuzzy DLs, showing the potential of our approach.

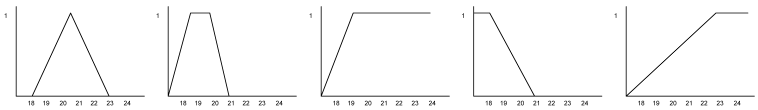
Example 1. The MCDM problem consists of four alternatives and five criteria for an electrical power dispatching system in the case of shortage of electrical power[2]. The four alternatives correspond to four regions of a city to which to give priority. The five criteria correspond to C_1 (Residential area), C_2 (Shopping centers), C_3 (Clubs and recreation centers), C_4 (Educational centers), and C_5 (Medical urgent care centers). We normalize the performance values according to $q_{ij} = w_j \cdot (a_{ij} / \sum_{l=1}^{n} a_{lj})$. The decision matrix together with the matrix of the q_{ij}'s is shown below:

		Criteria				
		0.20	0.1	0.3	0.35	0.05
	Alternatives	C_1	C_2	C_3	C_4	C_5
x_1	A_1	0.5	0.7	0.3	0.1	0.3
x_2	A_2	0.2	0.3	1.0	0.7	0.2
x_3	A_3	1.0	0.8	0.5	1.0	0.5
x_4	A_4	0.3	0.2	0.2	0.2	1.0

$$q_{ij} = \begin{pmatrix} 0.0455 & 0.0333 & 0.0391 & 0.0149 & 0.0073\\ 0.0182 & 0.0143 & 0.1304 & 0.1043 & 0.0049\\ 0.0909 & 0.0381 & 0.0652 & 0.1489 & 0.0122\\ 0.0273 & 0.0095 & 0.0261 & 0.0298 & 0.0244 \end{pmatrix}$$

[2] The use case is inspired by `http://med.ee.nd.edu/MED11/pdf/papers/t3-013.pdf`

Now, we further assume that each of the five criteria has *dynamic, time dependent* electricity demand on the day time (18-24) as depicted below (values are normalized to one).



C_1 C_2 C_3 C_4 C_5

We model this situation by assuming that we have in the KB the axioms:

$C_1 =$ ResidentialArea $\sqcap$ $\exists$hasDemand.tri(18, 20.50, 23)
$C_2 =$ ShoppingCenter $\sqcap$ $\exists$hasDemand.trap(17, 19, 20, 21)
$C_3 =$ ClubRicreationCenter $\sqcap$ $\exists$hasDemand.rs(17, 19)
$C_4 =$ EducationalCenter $\sqcap$ $\exists$hasDemand.ls(19, 21)
$C_5 =$ MedicalUrgentCareCenter $\sqcap$ $\exists$hasDemand.rs(23, 24)

where hasDemand is a functional concrete feature. We now define the four alternatives A_i as the following weighted concepts:

$$A_i = q_{i1} \cdot C_1 + q_{i2} \cdot C_2 + q_{i3} \cdot C_3 + q_{i4} \cdot C_4 \ , \text{ for } i = 1, 2, 3, 4 \ .$$

The *final rank value* of alternative A_i w.r.t. a knowledge base $\mathcal{K}$, denoted $rv(\mathcal{K}, A_i)$ is defined as $rv(\mathcal{K}, A_i) = bsd(\mathcal{K}, A_i)$, i.e. we compute its maximal satisfiability degree. It is thus, easily verified that this extents the WSM to the fuzzy DL case. Finally, the *optimal alternative* is $A^* = \arg\max_{A_i} rv(\mathcal{K}, A_i)$. Now, assume that we have a shortage of power between time 19-20. It can be verified that (the values have been computed using the fuzzy DL reasoner FUZZYDL [3]) $rv(\mathcal{K}, A_1) = 0.11625$, $rv(\mathcal{K}, A_2) = 0.25628$, $rv(\mathcal{K}, A_3) = 0.28856$, $rv(\mathcal{K}, A_4) = 0.07632$, and, thus, the ranking of the alternatives is $A_3 \succ A_2 \succ A_1 \succ A_4$ and the optimal alternative is $A^* = A_3$. □

Example 2. The following example is a simplified version of [4] and is about landfill siting. We have to select among two sites, $Site_1, Site_2$, according to two criteria (TI -Transportation Issues, and PN -Public Nuisance) and there are two experts (E_1, E_2). The decision matrix of the experts is shown below:

E_1		*Criteria*	
		0.48	0.52
Alternatives		C_1	C_2
x_1	A_1	$tri(0.6, 0.7, 0.8)$	$tri(0.9, 0.95, 1.0)$
x_2	A_2	$tri(0.6, 0.7, 0.8)$	$tri(0.4, 0.5, 0.6)$

E_2		*Criteria*	
		0.52	0.48
Alternatives		C_1	C_2
x_1	A_1	$tri(0.55, 0.6, 0.7)$	$tri(0.4, 0.45, 0.5)$
x_2	A_2	$tri(0.35, 0.4, 0.45)$	$tri(0.5, 0.55, 0.6)$

Note that this time, the performance of the alternatives is defined in terms fuzzy numbers. We may model the scenario as follows. For each expert $k = 1, 2$, for each alternative $i = 1, 2$ and for each criteria $j = 1, 2$, we define the concept

$$P_{ij}^k = \exists \mathsf{hasScore}.a_{ij}^k \ ,$$

where hasScore is a concrete feature and a_{ij}^k is, according to expert k, the performance of alternative i with respect to criteria j (a_{ij}^k is the fuzzy number in the matrix). Now, for each expert k and alternative i, we define the weighted concept

$$A_i^k = w_1^k \cdot P_{i1}^k + w_2^k \cdot P_{i2}^k \ ,$$

which takes into account also the weight w_j^k of expert k for criteria j. Finally, we combine the two experts outcome, by defining the weighted concept

$$A_i = 0.5 \cdot A_i^1 + 0.5 \cdot A_i^2 \ .$$

Note that we rate both experts equally (this may be changed, of course). The final rank value of alternative A_i w.r.t. a knowledge base $\mathcal{K}$ and the optimal alternative is determined as for Example 1. It can be verified (using again the fuzzy DL reasoner FUZZYDL [3]) that $rv(\mathcal{K}, A_1) = 0.26$ and $rv(\mathcal{K}, A_2) = 0.37$ and, thus, the ranking of the alternatives is $A_2 \succ A_1$ and the optimal alternative is $A^* = A_2$. □

5 Conclusions

We have made an initial step in addressing MCDM within fuzzy DLs and, thus, towards a fuzzy knowledge-assisted approach to decision making. Our aim here was exploratory on the argument and a more in depth investigations need to be addressed, of course.

There are several points that may be of interest for future research: (*i*) each alternative is indeed a fuzzy set and, so far, we order alternatives according to the best satisfiability degree. Other methods can be explored by relying on the fuzzy membership function of the alternatives, e.g. using defuzzification methods; (*ii*) fully exploit fuzzy numbers as performance and weight values in decision matrixes; (*iii*) for illustrative purposes, we have just considered a simple, though widely used, basic MCDM method, namely the weighted sum method. The MCDM literature (inclusive their fuzzy MCDM variants) is quite large, so it will be of interest whether and how other methods can be integrated within fuzzy DLs as well. E.g., for illustrative purposes, we considered the weighted sum method, though in general other fuzzy aggregation operators may be needed to combine the multiple performance values into an aggregated value (e.g., to cope with criteria interdependence/conflict); (*iv*) exploit the fact that we may express background/domain knowledge within fuzzy DLs (e.g., in Example 1 we may include an Urban Ontology to formalize the problem, which is part of larger GIS system; a similar argument applies to Example 2 as well); (*v*) fuzzy DLs are parametric with respect to t-norm, t-conorm, etc. Choosing, e.g., appropriate fuzzy connectors, as well as appropriate fuzzy aggregation operators, is clearly application specific and may bring to different results and, thus, these issue needs both theoretical and empirical investigations in our setting as well.

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